Blatt 1

VK 1.1.

(a) $8a - \left(a + \left((3a-2b) - (5a+3b)\right) - (-a+6)\right)$

$= 8a - \left(a + (3a-2b-5a-3b) + a - 6\right)$

$= 8a - (a+3a-2b-5a-3b+a-6) = 8a - (-5b-6)$

$= 8a + 5b + 6$

(b) $2a\left(a-(b-3a)\right) = 2a(a-b+3a) = 2a(4a-b) = 8a^2 - 2ab$

(c) $(3a+2b)(4a-3b)(5a-7b) = (12a^2 - 9ab + 8ab - 6b^2)(5a-7b)$

$= (12a^2 - ab - 6b^2)(5a-7b)$

$= 60a^3 - 84a^2b - 5a^2b + 7ab^2 - 30ab^2 + 42b^3$

$= 60a^3 - 89a^2b - 23ab^2 + 42b^3$

d) $(a+4)(a-2) - (a+2)(a-1) = a^2 - 2a + 4a - 8 - a^2 + a - 2a + 2$

$= a - 6$

e) $(1-a)(a-1) - 2(a+1)(a-2) = -(a-1)^2 - 2(a^2 - a - 2)$

$= -a^2 + 2a - 1 - 2a^2 + 2a + 4 = -3a^2 + 4a + 3$

VK 1.2

(a) $(a^2+b^2)^2 - (a^2-b^2)^2 = a^4 + 2a^2b^2 + b^4 - a^4 + 2a^2b^2 - b^4$

$= 4a^2b^2$

(b) $9a^4b^2 + 12a^2b + 4 = (3a^2b + 2)^2$

(c) $(\sqrt{ab} - 1)(-1 - \sqrt{ab})$

$= (-1 + \sqrt{ab})(-1 - \sqrt{ab}) = (-1)^2 - (\sqrt{ab})^2 = 1 - ab$

d) $a^2 + 2ab + b^2 - 4(a+b) + 4$

$= (a+b)^2 - 4(a+b) + 4 = (a+b-2)^2$

VK 1.3

(a) $-a^2 - a = -a(a+1)$

(b) $ab - ac - b + c = a(b-c) - (b-c)$

$= (a-1)(b-c)$

(c) $4a^2 + 20ab + 25b^2 - a^2 = (2a+5b)^2 - a^2$

$= (2a+5b+a)(2a+5b-a) = (3a+5b)(a+5b)$

(d) $(-a-1)(a-1) - (a^2-1) = -(a+1)(a-1) - (a+1)(a-1)$

$= -2(a+1)(a-1) = 2(a+1)(1-a)$

Vk 1.4.

$$\frac{2730}{5005} = \frac{2\cdot 5\cdot 3\cdot 7\cdot 13}{5\cdot 7\cdot 13\cdot 11} = \frac{6}{11}$$

$$\frac{69069}{138138} = \frac{3\cdot 7\cdot 11\cdot 13\cdot 23}{2\cdot 3\cdot 7\cdot 11\cdot 13\cdot 23} = \frac{1}{2}$$

oder:

$$\frac{2730}{5005} \overset{:5}{=} \frac{546}{1001} \overset{:7}{=} \frac{78}{143} \overset{:13}{=} \frac{6}{11}$$

$$\frac{69069}{138138} \overset{:3}{=} \frac{23023}{46046} \overset{:7}{=} \frac{3289}{6578} \overset{:11}{=} \frac{299}{598} \overset{:13}{=} \frac{23}{46} \overset{:23}{=} \frac{1}{2}$$

Vk 1.5.

(a)
$$\frac{6ab+9b}{6ab-6b} - \frac{6ab-4b}{6ab+6b} - \frac{10b^2}{12a^2b^2-12b^2}$$

$$= \frac{3b(2a+3)}{6b(a-1)} - \frac{2b(3a-2)}{6b(a+1)} - \frac{10b^2}{12b^2(a^2-1)}$$

$$= \frac{2a+3}{2(a-1)} - \frac{3a-2}{3(a+1)} - \frac{5}{6(a+1)(a-1)}$$

$$= \frac{3(2a+3)(a+1) - 2(3a-2)(a-1) - 5}{6(a+1)(a-1)}$$

$$= \frac{3(2a^2+5a+3) - 2(3a^2-5a+2) - 5}{6(a+1)(a-1)}$$

$$= \frac{6a^2+15a+9-6a^2+10a-4-5}{6(a+1)(a-1)} = \frac{25a}{6(a+1)(a-1)}$$

(b)
$$\frac{9a-b}{6a^2-2ab} - \frac{6a+b}{3ab-b^2} + \frac{1}{2b}$$

$$= \frac{9a-b}{2a(3a-b)} - \frac{6a+b}{b(3a-b)} + \frac{1}{2b}$$

$$= \frac{b(9a-b) - 2a(6a+b) + a(3a-b)}{2ab(3a-b)}$$

$$= \frac{9ab-b^2-12a^2-2ab+3a^2-ab}{2ab(3a-b)}$$

$$= \frac{6ab-9a^2-b^2}{2ab(3a-b)} = -\frac{9a^2-6ab+b^2}{2ab(3a-b)}$$

$$= -\frac{(3a-b)^2}{2ab(3a-b)} = \frac{b-3a}{2ab}$$

zu VK 1.5

(c) $$\left(\frac{2}{a}+\frac{3}{b}\right)\left(\frac{a}{2}-\frac{b}{3}\right) = \frac{2}{a}\cdot\frac{a}{2} - \frac{2b}{3a} + \frac{3a}{2b} - \frac{3}{b}\cdot\frac{b}{3}$$

$$= \not{1} - \frac{2b}{3a} + \frac{3a}{2b} - \not{1}$$

$$= \frac{9a^2-4b^2}{6ab} \quad \left(= \frac{(3a+2b)(3a-2b)}{6ab} \right)$$

d)

$$\left(\frac{a+b}{b}+\frac{a+b}{a}\right):\left(\frac{1}{a}+\frac{1}{b}\right) = (a+b)\left(\frac{1}{b}+\frac{1}{a}\right):\left(\frac{1}{a}+\frac{1}{b}\right) = a+b$$

(e) $$\frac{\frac{a}{1-a}+\frac{a+1}{a}}{\frac{a-1}{a}-\frac{a}{a+1}} = \frac{a^2+(a+1)(1-a)}{a(1-a)} : \frac{(a-1)(a+1)-a^2}{a(a+1)}$$

$$= \frac{a^2+1-a^2}{\not{a}(1-a)} \cdot \frac{\not{a}(a+1)}{a^2-1-a^2} = \frac{a+1}{a+1}$$

(5) (6)

(1) $$\frac{1}{a-\frac{a}{1-\frac{a}{a-b}}} = \frac{1}{a-\frac{a}{\frac{a-b-a}{a-b}}}$$

$$= \frac{1}{a+\frac{a(a-b)}{b}} = \frac{1}{\frac{ab+a^2-ab}{b}} = \frac{b}{a^2}$$

VK 1 6

(a) $$\frac{a-\sqrt{a}\cdot b}{b-\sqrt{a}} = \frac{\sqrt{a}\cdot\sqrt{a}-\sqrt{a}\cdot b}{b-\sqrt{a}} = \frac{\sqrt{a}(\sqrt{a}-b)}{b-\sqrt{a}} = -\sqrt{a}$$

$$\frac{a-\sqrt{a}\cdot b}{b-\sqrt{a}} = \frac{(a-\sqrt{a}b)(b+\sqrt{a})}{b^2-a}$$

$$= \frac{ab+a\sqrt{a}-b^2\sqrt{a}-ab}{b^2-a} = \frac{\sqrt{a}(a-b^2)}{b^2-a} = -\sqrt{a}$$

(b) $$\frac{ax+\frac{x}{b}-\frac{a}{y}-\frac{1}{by}}{\frac{1}{b}+a} \overset{\cdot by}{=} \frac{abxy+xy-ab-1}{y+aby}$$

$$= \frac{ab(xy-1)+(xy-1)}{y(1+ab)} = \frac{(xy-1)(ab+1)}{y(1+ab)} = \frac{xy-1}{y}$$

(c) $$\frac{(a+b)^4-(a-b)^4}{a^2+b^2} = \frac{((a+b)^2+(a-b)^2)((a+b)^2-(a-b)^2)}{a^2+b^2}$$

$$= \frac{(a^2+2ab+b^2+a^2-2ab+b^2)(a^2+2ab+b^2-a^2+2ab-b^2)}{a^2+b^2}$$

$$= \frac{2(a^2+b^2)\cdot 4ab}{a^2+b^2} = 8ab$$

VK 1.7. $$1-\frac{25a^2-36b^2}{6b-5a} = 1-\frac{(5a+6b)(5a-6b)}{6b+5a} = 1+5a+6b$$

VK 2.1

(a) $a(x+1)(ax+b)+b(a+bx)(1-x)=x^2(a-b)(a+b)$

$$a(ax^2+bx+ax+b)+b(a-ax+bx-bx^2)=x^2(a^2-b^2)$$

$$a^2x^2+abx+a^2x+ab+ab-abx+b^2x-b^2x^2=a^2x^2-b^2x^2$$

$$(a^2+b^2)x+2ab=0$$

$$\Leftrightarrow \quad (a^2+b^2)x=-2ab$$

falls $a^2+b^2=0 \Rightarrow a=0 \wedge b=0 \Rightarrow \mathbb{L}=\mathbb{R}$

falls $a^2+b^2\neq 0 \Rightarrow x=-\frac{2ab}{a^2+b^2}$

(b) $\frac{ax^2-bx+1}{a}=\frac{bx^2-ax+1}{b} \quad |\cdot ab \qquad ; a\neq 0 \wedge b\neq 0$

$$abx^2-b^2x+b=abx^2-a^2x+a$$

$$(a^2-b^2)x=a-b \quad \Leftrightarrow \quad \boxed{(a+b)(a-b)x=a-b}$$

1. Fall: $a^2-b^2=0 \Leftrightarrow (a+b)(a-b)=0$
   1) $a+b=0 \Rightarrow a-b\neq 0 \Rightarrow (a+b)x=1 \Rightarrow \mathbb{L}=\emptyset$
   2) $a-b=0 \Rightarrow \mathbb{L}=\mathbb{R}$

2. Fall: $a^2-b^2\neq 0 \Rightarrow x=\frac{a-b}{a^2-b^2}=\frac{1}{a+b}$

<u>oder so:</u>

falls $a-b=0 \Rightarrow \mathbb{L}=\mathbb{R}$

falls $a-b\neq 0 \Rightarrow (a+b)x=1$

falls $a+b=0 \Rightarrow \mathbb{L}=\emptyset$

falls $a+b\neq 0 \Rightarrow x=\frac{1}{a+b}$

(c) $\frac{x-\sqrt{a}}{x-\sqrt{b}} - \frac{x-\sqrt{a}}{x+\sqrt{b}} = 0$ $\quad |x| \neq \sqrt{b}$ $\quad$ (8)

$a, b \geq 0$

$$(x-\sqrt{a})(x+\sqrt{b}) - (x-\sqrt{a})(x-\sqrt{b}) = 0$$

$$(x-\sqrt{a})(2\sqrt{b}) = 0$$

$$(x-\sqrt{a})\sqrt{b} = 0$$

falls $b = 0 \Rightarrow \mathbb{L} = \mathbb{R} \setminus \{\pm\sqrt{b}\}$

falls $b \neq 0$ $(x-\sqrt{a}=0)$ $\begin{cases} 1) \; a = b \Rightarrow \mathbb{L} = \emptyset \\ 2) \; a \neq b \Rightarrow x = \sqrt{a} \end{cases}$

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d) $\frac{\frac{1}{a} - \frac{1}{x}}{\frac{1}{a} + \frac{1}{x}} = \frac{a - \frac{1}{x}}{a + \frac{1}{x}}$ $\quad a \neq 0 \; ; \; x \neq 0$

$x \neq -a \; ; \; x \neq -\frac{1}{a}$

d.h. $x \notin \{0; -a; -\frac{1}{a}\}$

mit ax erw.: $\frac{x-a}{x+a} = \frac{ax-1}{ax+1}$ mit x erw.

$$(x-a)(ax+1) = (ax-1)(x+a)$$

$$\cancel{ax^2} + x - a^2x - \cancel{a} = \cancel{ax^2} + a^2x - x - \cancel{a} \iff 0 = 2a^2x - 2x$$

$$(2a^2-2)x = 0 \iff (a^2-1)x = 0$$

falls $a^2 - 1 = 0$ $\quad$ 1) $a = 1 \Rightarrow \mathbb{L} = \mathbb{R} \setminus \{0; -1\}$

2) $a = -1 \Rightarrow \mathbb{L} = \mathbb{R} \setminus \{0; 1\}$

falls $a^2 - 1 \neq 0 \Rightarrow \mathbb{L} = \emptyset$

VK 2.2.

$$\text{I.}\quad x - y = 6 \quad \Rightarrow x = y + 6$$
$$\text{II.}\quad x^2 - y^2 = 180$$

$$\Rightarrow \text{II} \quad (y+6)^2 - y^2 = 180$$
$$y^2 + 12y + 36 - y^2 = 180$$
$$12y = 144 \quad | : 12 \quad \Rightarrow y = 12$$
$$\Rightarrow x = y + 6 = 18$$

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oder so

$$\text{II} \quad (x+y)(x-y) = 180$$
$$\Leftrightarrow \quad 6(x+y) = 180$$
$$\Rightarrow \quad x + y = 30$$

$$\text{I} \quad x - y = 6$$
$$\text{II} \quad x + y = 30$$

$$\text{I} + \text{II} \quad 2x = 36 \Rightarrow x = 18 \quad \Rightarrow y = 12$$

VK 2.3.

$$(a)\ \text{I.}\ \frac{3}{x} + \frac{8}{y} = 3 \quad \Rightarrow 5\text{I}: \frac{15}{x} + \frac{40}{y} = 15$$
$$\text{II.}\ \frac{15}{x} - \frac{4}{y} = 4$$

$$5\text{I.}\quad \frac{15}{x} + \frac{40}{y} = 15$$
$$\text{II.}\quad \frac{15}{x} - \frac{4}{y} = 4$$

$$5\text{I} - \text{II} \quad \frac{44}{y} = 11 \quad \Rightarrow y = \frac{44}{11} = 4$$

$$\Rightarrow \frac{3}{x} + 2 = 3 \Rightarrow \frac{3}{x} = 1 \Rightarrow x = 3$$

Alternative Substitution $a = \frac{1}{x}$ und $b = \frac{1}{y}$

$$\text{I.}\ 3a + 8b = 3$$
$$\text{II.}\ 15a - 4b = 4$$

$$5\text{I} - \text{II} \quad 44b = 11 \Rightarrow b = \frac{1}{4} \Rightarrow y = 4$$
$$\text{I} \Rightarrow 3a + 2 = 3 \Rightarrow 3a = 1 \Rightarrow a = \frac{1}{3}$$
$$\Rightarrow x = 3$$

zu VK 2.3.

$a+b \neq 0$; $a-b \neq 0$

d.h. $|a| \neq b$

(b)

$$\text{I.}\quad (a-b)x + y = \frac{a+b+1}{a+b}$$

$$\text{II.}\quad x + (a+b)y = \frac{a-b+1}{a-b}$$

$$(a+b)\,\text{I.}\quad (a^2-b^2)x + (a+b)y = a+b+1$$

$$\text{II}\quad x + (a+b)y = \frac{a-b+1}{a-b}.$$

$$(a+b)\,\text{I} - \text{II}:\quad (a^2-b^2-1)x = a+b+1 - \frac{a-b+1}{a-b}$$

$$= \frac{(a+b+1)(a-b) - a+b-1}{a-b}$$

$$= \frac{a^2 - ab + ab - b^2 + a - b - a + b - 1}{a-b}$$

$$\boxed{(a^2-b^2-1)x = \frac{a^2-b^2-1}{a-b}}$$

1. Fall: $a^2-b^2-1=0 \Rightarrow x \in \mathbb{R}$ beliebig

$$\Rightarrow y = \frac{a+b+1}{a+b} - (a-b)x$$

$$\mathbb{L} = \left\{ \left(x ; \frac{a+b+1}{a+b} - (a-b)x \right) \mid x \in \mathbb{R} \right\}$$

2. Fall: $a^2-b^2-1 \neq 0 \Rightarrow x = \frac{1}{a-b} \Rightarrow y = \frac{a+b+1}{a+b} - 1 = \frac{1}{a+b}$

$$\mathbb{L} = \left\{ \left(\frac{1}{a-b} ; \frac{1}{a+b} \right) \right\}$$

I. $x+y = x^2 - y^2$

II. $(x+4)^2 - (y-4)^2 = 99$

I. $x+y = x^2 - y^2 = (x+y)(x-y)$

II. $x^2 + 8x + 16 - y^2 + 8y - 16 = 99$

$x^2 - y^2 + 8(x+y) = 99$

I eingesetzt in II:

$x+y + 8(x+y) = 99$

$9(x+y) = 99 \Rightarrow x+y = 11$

I $\Rightarrow x-y = 1$

$2x = 12 \Rightarrow x = 6$

$y = 5$

Alternative: Substitution $a = x+y$; $b = x-y$

I $a = ab$

II. $ab + 8a = 99$

I. $-ab + a = 0$

II $ab + 8a = 99$

I + II $9a = 99 \Rightarrow a = 11$

aus I $\Rightarrow b = 1$

also

I. $x+y = 11$

II. $x-y = 1$

I + II $2x = 12 \Rightarrow x = 6$

$y = 5$

VK 2.5.

(a)
$$\frac{a^{3n-x}\cdot b^{2n+x}}{a^{n+2x}\cdot b^{2n-x}}\cdot\frac{x^{3n+2}\cdot y^{2n-1}}{x^{2n-3}\cdot y^{n+1}}$$

$$= a^{3n-x-n-2x}\cdot b^{2n+x-2n+x}\cdot x^{3n+2-2n+3}\cdot y^{2n-1-n-1}$$

$$= a^{2n-3x}\cdot b^{2x}\cdot x^{n+5}\cdot y^{n-2}$$

(b)
$$\frac{a^{-2}\cdot x^{-4}\cdot y^{-6}}{b^{3}\cdot c^{-4}\cdot z^{-5}}\cdot\frac{c^{-5}\cdot y^{6}\cdot z^{-7}}{a^{-3}\cdot b^{-5}\cdot x^{-3}}$$

$$= a^{-2+3}\, b^{-3+5}\cdot c^{-5+4}\cdot x^{-4+3}\cdot y^{6-6}\cdot z^{-7+5}$$

$$= a\, b^{2}\, c^{-1}\, x^{-1}\, z^{-2} = \frac{ab^{2}}{cxz^{2}}$$

(c)
$$\frac{1}{x+\sqrt{x^2+a^2}}+\frac{x}{\left(x+\sqrt{x^2+a^2}\right)\cdot\sqrt{x^2+a^2}}$$

$$= \frac{\left(\sqrt{x^2+a^2}+x\right)}{\left(x+\sqrt{x^2+a^2}\right)\cdot\sqrt{x^2+a^2}} = \frac{1}{\sqrt{x^2+a^2}}$$

VK 2.6.

(a)

$$\sqrt{\frac{a^2}{8}+\sqrt{\left(\frac{a^2}{8}\right)^2+\frac{a^4}{8}}} = \left(\frac{a^2}{8}+\left(\frac{a^4}{64}+\frac{a^4}{8}\right)^{1/2}\right)^{1/2}$$

$$= \left(\frac{a^2}{8}+\left(\frac{a^4+8a^4}{64}\right)^{1/2}\right)^{1/2} = \left(\frac{a^2}{8}+\left(\frac{9a^4}{64}\right)^{1/2}\right)^{1/2}$$

$$= \left(\frac{a^2}{8}+\frac{3a^2}{8}\right)^{1/2} = \left(\frac{4a^2}{8}\right)^{1/2} = \left(\frac{a^2}{2}\right)^{1/2} = \frac{a}{2^{1/2}}$$

$$= a\cdot 2^{-\frac{1}{2}}$$

(b)

$$\sqrt[3]{a^3\sqrt{a^2\cdot\sqrt[5]{a^8\cdot\sqrt[4]{a^3}}}} = \sqrt[3]{a^3\sqrt{a^2\left(a^8\cdot a^{3/4}\right)^{1/5}}}$$

$$= \sqrt[3]{a^3\sqrt{a^2\left(a^{35/4}\right)^{1/5}}} = \sqrt[3]{a^3\sqrt{a^2 a^{7/4}}}$$

$$= \sqrt[3]{a^3\left(a^{15/4}\right)^{1/2}} = \sqrt[3]{a^3\cdot a^{15/8}}$$

$$= \left(a^{39/8}\right)^{1/3} = a^{13/8}$$

VK 2.7.

(a)

$$\frac{ab}{\sqrt[7]{a^2b^3}} = \frac{\sqrt[7]{a^7b^7}}{\sqrt[7]{a^2b^3}} = \sqrt[7]{\frac{a^7b^7}{a^2b^3}} = \sqrt[7]{a^5b^4}$$

(b)

$$\frac{6}{\sqrt{5}+1} = \frac{6(\sqrt{5}-1)}{(\sqrt{5}+1)(\sqrt{5}-1)} = \frac{6(\sqrt{5}-1)}{5-1} = \frac{6}{4}(\sqrt{5}-1) = \frac{3}{2}(\sqrt{5}-1)$$

(c)

$$\frac{1+\sqrt{2}+\sqrt{3}}{1+\sqrt{2}-\sqrt{3}} = \frac{(1+\sqrt{2}+\sqrt{3})^2}{(1+\sqrt{2})^2-3} = \frac{(\ldots)^2}{1+2\sqrt{2}+2-3} = \frac{(\ldots)^2}{2\sqrt{2}}$$

$$= \frac{(\ldots)^2}{2\cdot 2/\sqrt{2}} = \frac{(\ldots)^2}{4/\sqrt{2}} = \frac{\sqrt{2}(\ldots)^2}{4}$$

VK 7.1. komplexe $x_{1/2} = \pm\sqrt{-9} = \pm 3i$ (14)

(a) $3x^2+27=0 \iff x^2+9=0 \iff x^2=-9$

$\mathbb{L}=\emptyset$

(b) $(x+\frac{1}{2})(x-\frac{1}{2})=\frac{5}{16} \iff x^2-\frac{1}{4}=\frac{5}{16}$

$x^2=\frac{5}{16}+\frac{1}{4}=\frac{5}{16}+\frac{4}{16}=\frac{9}{16}$

$x_1=\frac{3}{4}$; $x_2=-\frac{3}{4}$; $\mathbb{L}=\{\pm\frac{3}{4}\}$

(c) $(x-\sqrt{7})(x-\sqrt{5})=0$

$x-\sqrt{7}=0$ oder $x-\sqrt{5}=0$

$\Rightarrow x_1=\sqrt{7}$ $\Rightarrow x_2=\sqrt{5}$

$\mathbb{L}=\{\sqrt{5};\sqrt{7}\}$

(d) $x^2+4x+2=0 \iff (x+2)^2-4+2=0$

$\iff (x+2)^2=2$

$\iff x_{1/2}=\pm\sqrt{2}-2$

$x_1=\sqrt{2}-2$; $x_2=-\sqrt{2}-2$

oder so: $x_{1/2}=\frac{-4\pm\sqrt{16-8}}{2}=\frac{-4\pm\sqrt{8}}{2}=\frac{-4\pm2\sqrt{2}}{2}$

$=-2\pm\sqrt{2}$

zu VK 3.1.

(e) $(3x-5)^2 - (2x+5)^2 = 0$

1. Möglichkeit: $(3x-5)^2 = (2x+5)^2$

1. Fall: $3x - 5 = 2x + 5 \Rightarrow x_1 = 10$

2. Fall: $3x - 5 = -(2x+5) = -2x - 5 \Big\} \Rightarrow 5x = 0 \Rightarrow x_2 = 0$

2. Möglichkeit: oder auch so: $(3x-5+2x+5)(3x-5-2x-5) = 0 \Leftrightarrow 5x(x-10) = 0$

$$9x^2 - 30x \cancel{+25} - 4x^2 - 20x \cancel{-25} = 0$$

$$5x^2 - 50x = 0$$

$$5x(x-10) = 0 \Rightarrow x_1 = 10 ,\ x_2 = 0$$

(f) $7x^2 + 32x = 84 \Leftrightarrow 7x^2 + 32x - 84$

$$x_{1/2} = \frac{-32 \pm \sqrt{32^2 + 28\cdot 84}}{14} = \frac{-32 \pm \sqrt{3376}}{14}$$

$$= \frac{-32 \pm \sqrt{16\cdot 211}}{14} = \frac{-32 \pm 4\sqrt{211}}{14} = \frac{-16 \pm 2\sqrt{211}}{7}$$

$$x_1 = \frac{1}{7}\left(-16 + 2\sqrt{211}\right) \approx 1{,}865$$

$$x_2 = \frac{1}{7}\left(-16 - 2\sqrt{211}\right) \approx -6{,}436$$

VK 3.2.

(a) $12x^2 - 34ax + 10a^2 = 0 \quad | :2$

$6x^2 - 17ax + 5a^2 = 0$; falls $a = 0 \Rightarrow x = 0$

nun $a \neq 0$: $x_{1/2} = \frac{17a \pm \sqrt{289a^2 - 120a^2}}{12} = \frac{17a \pm \sqrt{169a^2}}{12}$

$$= \frac{17a \pm 13a}{12} \Rightarrow x_1 = \frac{30a}{12} = \frac{5a}{2}$$

$$x_2 = \frac{4a}{12} = \frac{a}{3}$$

zu VK 3.2.

(b) $abx^2 - (a^2+b^2)x + ab = 0$

1. Fall: $a = 0 \wedge b = 0 \Rightarrow \mathbb{L} = \mathbb{R}$

2. Fall: $(a = 0 \wedge b \neq 0) \vee (a \neq 0 \wedge b = 0)$

$\Rightarrow \quad -(a^2+b^2)x = 0 \Rightarrow x = 0$

3. Fall: $a \neq 0 \wedge b \neq 0$

$$x_{1,2} = \frac{a^2+b^2 \pm \sqrt{(a^2+b^2)^2 - 4a^2b^2}}{2ab}$$

$$= \frac{a^2+b^2 \pm \sqrt{(a^2+b^2+2ab)(a^2+b^2-2ab)}}{2ab}$$

$$= \frac{a^2+b^2 \pm \sqrt{(a+b)^2(a-b)^2}}{2ab} = \frac{a^2+b^2 \pm (a+b)(a-b)}{2ab}$$

$$= \frac{a^2+b^2 \pm (a^2-b^2)}{2ab} \Rightarrow x_1 = \frac{a^2+b^2+a^2-b^2}{2ab} = \frac{2a^2}{2ab} = \frac{a}{b}$$

$$x_2 = \frac{a^2+b^2-a^2+b^2}{2ab} = \frac{2b^2}{2ab} = \frac{b}{a}$$

Zu VK 3.2.

(c) $x^2 + \frac{1}{2}bx - \frac{1}{2}b^2 = 0 \quad | \cdot 2$

$\iff \quad 2x^2 + bx - b^2 = 0$

$x_{1/2} = \frac{-b \pm \sqrt{b^2 + 8b^2}}{4} = \frac{-b \pm \sqrt{9b^2}}{4}$

$= \frac{-b \pm 3b}{4}$

$\Rightarrow \quad x_1 = \frac{-b + 3b}{4} = \frac{2b}{4} = \frac{b}{2}$

$x_2 = \frac{-b - 3b}{4} = \frac{-4b}{4} = -b$

(d) $x - \frac{1}{x} = \frac{a}{b} - \frac{b}{a} \quad | \cdot abx \quad ; \quad a \neq 0 \; ; \; b \neq 0$

$abx^2 - ab = a^2x - b^2x = (a^2 - b^2)x$

$abx^2 - (a^2 - b^2)x - ab = 0$

$x_{1/2} = \frac{(a^2 - b^2) \pm \sqrt{(a^2 - b^2)^2 + 4a^2b^2}}{2ab}$

$= \frac{a^2 - b^2 \pm \sqrt{a^4 - 2a^2b^2 + b^4 + 4a^2b^2}}{2ab}$

$= \frac{a^2 - b^2 \pm \sqrt{a^4 + 2a^2b^2 + b^4}}{2ab}$

$= \frac{a^2 - b^2 \pm \sqrt{(a^2 + b^2)^2}}{2ab} = \frac{a^2 - b^2 \pm (a^2 + b^2)}{2ab}$

$\Rightarrow \quad x_1 = \frac{a^2 - b^2 + a^2 + b^2}{2ab} = \frac{2a^2}{2ab} = \frac{a}{b}$

$x_2 = \frac{a^2 - b^2 - a^2 - b^2}{2ab} = -\frac{2b^2}{2ab} = -\frac{b}{a}$

VK 3.3. (18)

$$\left(\frac{a-x}{x-b}\right)^2 = 8\left(\frac{a-x}{x-b}\right) - 15 \quad ; \quad x \neq b, \; a \neq b$$

Substitution $u = \frac{a-x}{x-b} \quad | \cdot (x-b)$

$$u(x-b) = a-x \iff ux - ub = a-x \quad | +x+ub$$

$$\iff ux + x = a + ub$$

$$(u+1)x = a + ub \quad | : (u+1)$$

$$x = \frac{a+ub}{u+1}$$

Man erhält:

$$u^2 = 8u - 15 \iff u^2 - 8u + 15 = 0$$

$$u_{1/2} = \frac{8 \pm \sqrt{64-60}}{2} = \frac{8 \pm 2}{2} = 4 \pm 1$$

$$\Rightarrow u_1 = 5 \; ; \; u_2 = 3$$

$$\Rightarrow x_1 = \frac{a+5b}{6} \; ; \; x_2 = \frac{a+3b}{4}$$

falls $a = b \Rightarrow (-1)^2 = 8(-1) - 15$

$$1 = -8 - 15 = -23 \; (\neq)$$

$$\Rightarrow \mathbb{L} = \emptyset$$

VK 3.4.

Natürliche Zahlen x; $x+1$; $x+2$

$$(x+2)^2 = x^2 + (x+1)^2$$

$$x^2 + 4x + 4 = x^2 + x^2 + 2x + 1$$

$$x^2 - 2x - 3 = 0 \; ; \quad x = \frac{2 + \sqrt{4+12}}{2} = \frac{2+4}{2} = 3$$

Probe: 3; 4; 5

$$5^2 = 3^2 + 4^2 \iff 25 = 9 + 16 \checkmark$$

VK 3.5. Sei x der Durchmesser des Kreises

$$\left(\frac{x+3}{2}\right)^2 \pi = 2 \cdot \left(\frac{x}{2}\right)^2 \pi$$

$$\frac{(x+3)^2}{4} = 2 \cdot \frac{x^2}{4} = \frac{x^2}{2} \quad | \cdot 4$$

$$(x+3)^2 = 2x^2$$

$$x^2 + 6x + 9 = 2x^2 \iff x^2 - 6x - 9 = 0$$

$$x = \frac{6 + \sqrt{36+36}}{2} = \frac{6 + \sqrt{2 \cdot 36}}{2} = \frac{6 + 6\sqrt{2}}{2}$$

$$x = 3 + 3\sqrt{2} \approx 7{,}243$$

$$x = 3(1 + \sqrt{2})$$

VK 3.6.

(a) $10x^4 - 21 = x^2 \iff 10x^4 - x^2 - 21 = 0$

Substituiere $u := x^2$

$$10u^2 - u - 21 = 0$$

$$u = \frac{1 + \sqrt{1 + 840}}{20} = \frac{1 + 29}{20} = \frac{30}{20} = \frac{3}{2}$$

$$\Rightarrow x_1 = \sqrt{\tfrac{3}{2}} \;;\; x_2 = -\sqrt{\tfrac{3}{2}} \qquad ; \quad \sqrt{\tfrac{3}{2}} = \tfrac{1}{2}\sqrt{6}$$

(b) $x^3 + x - 2 = 0$

Durch Erraten : $x_1 = 1$

$$(x^3 + x - 2) : (x - 1) = x^2 + x + 2 \qquad \Rightarrow x^3 + x - 2 = (x^2 + x + 2)(x - 1) = 0$$

$-(x^3 - x^2)$

$x^2 + x - 2$

$-(x^2 - x)$

$2x - 2$

$-(2x - 2)$

$- - - -$

$$x^2 + x + 2 = 0$$

$$x_{2/3} = \frac{-1 \pm \sqrt{1 - 8}}{2} \quad \text{keine Lösung in } \mathbb{R}$$

da $-7 < 0$

$$\Rightarrow \mathbb{L} = \{1\} .$$

mit Horner-Schema!

	x^3	x^2	x	x^0
	1	0	1	-2
$x = 1$	0	1	1	2
	1	1	2	0

zu VK 3.6

(c) $x + 1 = \sqrt{2x^2 + \frac{1}{2}x + \frac{3}{2}}$

$(x+1)^2 = 2x^2 + \frac{x}{2} + \frac{3}{2}$

$x^2 + 2x + 1 = 2x^2 + \frac{x}{2} + \frac{3}{2}$

$2x + 1 = x^2 + \frac{x}{2} + \frac{3}{2} \quad / \cdot 2$

$4x + 2 = 2x^2 + x + 3$

$2x^2 - 3x + 1 = 0$

$x_{1/2} = \frac{3 \pm \sqrt{9 - 8}}{4} = \frac{3 \pm 1}{4}$

$\Rightarrow x_1 = 1 \quad ; \quad x_2 = \frac{1}{2}$

Probe:

$x = 1: \quad 1 + 1 = \sqrt{2 + \frac{1}{2} + \frac{3}{2}} = \sqrt{2+2}$ ✓

$x = \frac{1}{2}: \quad \frac{1}{2} + 1 = \sqrt{2 \cdot \frac{1}{4} + \frac{1}{4} + \frac{3}{2}} = \sqrt{\frac{2+1+6}{4}} = \frac{3}{2}$ ✓

Es kann auch sein, daß bei einer Wurzelgleichung ein x nicht Lösung der Ausgangsgleichung ist:

z. B. $x = -\sqrt{x}$

$\Rightarrow x^2 = x \quad \Rightarrow x^2 - x = 0$

$\Leftrightarrow x(x-1) = 0$

$\Rightarrow x_1 = 0 \; ; \; x_2 = 1$

$x_1 = 0$ Lösung, denn $0 = -\sqrt{0}$ ✓

$x_2 = 1$ keine Lösung, denn $1 = -\sqrt{1} = -1$ (↯)

VK 3.7.

(a) $\lg \sqrt{\frac{1}{100}} = \lg \frac{1}{10} = \lg 10^{-1} = -1$

(b) $\ln \sqrt{e^{3(\ln e^2 + \ln e^6)}} = \ln \sqrt{e^{3(2+6)}} = \ln \sqrt{e^{24}}$

$= \ln e^{12} = 12$

(c) $\log\left(2\sqrt{3 \cdot \sqrt[3]{a^2 b \cdot \sqrt[4]{ac^2}}}\right)$

$= \log 2 + \frac{1}{2} \log\left(3 \cdot \sqrt[3]{a^2 b \cdot \sqrt[4]{ac^2}}\right)$

$= \log 2 + \frac{1}{2} \log 3 + \frac{1}{6} \log\left(a^2 b \cdot \sqrt[4]{ac^2}\right)$

$= \log 2 + \frac{1}{2} \log 3 + \underline{\frac{1}{3} \log a} + \frac{1}{6} \log b + \underline{\frac{1}{24} \log a} + \frac{1}{12} \log c$

$= \log 2 + \frac{1}{2} \log 3 + \frac{3}{8} \log a + \frac{1}{6} \log b + \frac{1}{12} \log c$

VK 3.8.

(a) $|x+2| = 7$

1. Fall: $x+2 \geq 0 \Rightarrow x \geq -2$

$\Rightarrow x+2 = 7 \Rightarrow x = 5$

2. Fall: $x+2 < 0 \Rightarrow x < -2$

$\Rightarrow -x-2 = 7 \Rightarrow -x = 9 \Rightarrow x = -9$

$\mathbb{L} = \{5; -9\}$

Gesucht sind die Zahlen, die von -2 den Abstand 7 haben.

(b) $|x+3| = |3x-4|$

1. Fall: $x+3 \geq 0$ und $3x-4 \geq 0$; d.h. $x \geq -3$ und $x \geq \frac{4}{3}$

$x+3 = 3x-4 \Leftrightarrow 2x = 7 \Rightarrow x = \frac{7}{2}$

2. Fall: $x \geq -3$ und $x < \frac{4}{3}$

$x+3 = 4-3x \Rightarrow 4x = 1 \Rightarrow x = \frac{1}{4}$

3. Fall: $x < -3$ und $x \geq \frac{4}{3}$ ↯

4. Fall: $x < -3$ und $x < \frac{4}{3}$

$-(x+3) = -(3x-4) \Leftrightarrow x+3 = 3x-4$
wie 1. Fall

$\mathbb{L} = \{\frac{7}{2}; \frac{1}{4}\}$

zu VK 3.8.

(c) $|x^2+6x+5| = 5$

zunächst $x^2+6x+5 = 0$

$$x_{1/2} = \frac{-6 \pm \sqrt{36-20}}{2} = \frac{-6 \pm 4}{2} = -3 \pm 2$$

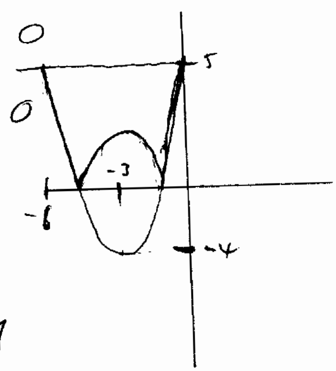
$x_1 = -1$; $x_2 = -5$

$\Rightarrow x^2+6x+5 = (x+1)(x+5)$

1. Fall: $x^2+6x+5 \geq 0$ d.h. $x \geq -1$ oder $x \leq -5$

$x^2+6x+5 = 5 \iff x^2+6x = 0$

$\iff x(x+6) = 0$



$x_1 = 0$, $x_2 = -6$

2. Fall: $x^2+6x+5 < 0$, d.h. $-5 < x < -1$

$x^2+6x+5 = -5 \iff x^2+6x+10 = 0$

$$x_{1/2} = \frac{-6 \pm \sqrt{36-40}}{2}$$ keine reelle Lösung!

$\Rightarrow \mathbb{L} = \{0; -6\}$

VK 3.9.

(a) $5 - 7x \leq 3x - 10 \iff 15 \leq 10x$

$\iff \frac{3}{2} \leq x$

$\Rightarrow \mathbb{L} = [\frac{3}{2}; \infty[$

(b) $\frac{5x+3}{x} < -2$

1. Fall: $x > 0$

$5x+3 < -2x \iff 7x < -3$

$x < -\frac{3}{7}$ ↯

geht nicht!

2. Fall: $x < 0$

$5x+3 > -2x \iff 7x > -3$

$\iff x > -\frac{3}{7}$

$\Rightarrow \mathbb{L} =]-\frac{3}{7}; 0[$

(c) $\frac{x+3}{x-7} < 0$

1. Fall: $x > -3$ und $x < 7$, d.h. $-3 < x < 7$

2. Fall: $x < -3$ und $x > 7$ ↯ geht nicht

$\mathbb{L} =]-3; 7[$

(d) $|2-4x| \geq 1$

1. Fall: $2-4x > 0 \Rightarrow 2 > 4x \Rightarrow x < \frac{1}{2}$

$2-4x \geq 1 \Rightarrow 1 \geq 4x \Rightarrow x \leq \frac{1}{4}$

$\Rightarrow \mathbb{L}_1 = \left]-\infty ; \frac{1}{4}\right]$

2. Fall: $x > \frac{1}{2}$

$4x-2 \geq 1 \Rightarrow 4x \geq 3 \Rightarrow x \geq \frac{3}{4}$

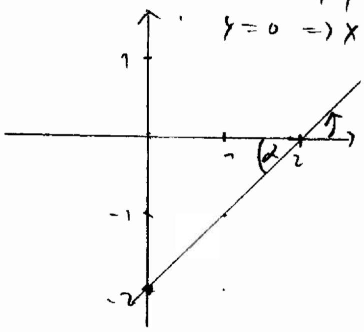
$\Rightarrow \mathbb{L}_2 = \left[\frac{3}{4} ; \infty\right[$

$\Rightarrow \mathbb{L} = \mathbb{L}_1 \cup \mathbb{L}_2 = \left]-\infty ; \frac{1}{4}\right] \cup \left[\frac{3}{4} ; \infty\right[$

VK 4.1

(a) $y = x - 2$

$x = 0 \Rightarrow y = -2$
$y = 0 \Rightarrow x = 2$



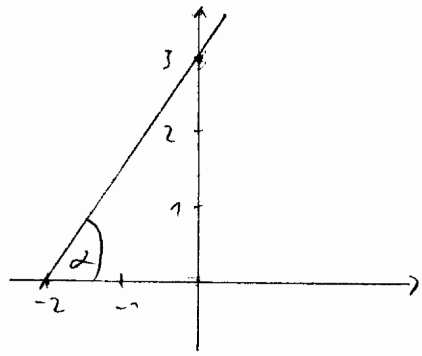
$m = 1$

$\alpha = \tan^{-1}(1) = 45°$

(b) $4y - 6x = 12$

$\left(y = \frac{6}{4} x + \frac{12}{4} = \frac{3}{2} x + 3 \right)$

$x = 0 \Rightarrow y = 3$
$y = 0 \Rightarrow x = -2$

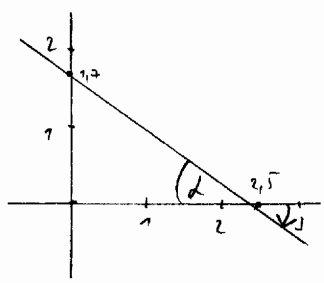


$m = \frac{3}{2} \Rightarrow \alpha = \tan^{-1}\left(\frac{3}{2}\right) = 56{,}31°$

(c) $y = \frac{1}{3}(5 - 2x)$

$= -\frac{2}{3} x + \frac{5}{3}$

$x = 0 \Rightarrow y = \frac{5}{3} = 1{,}7$
$y = 0 \Rightarrow x = \frac{5}{2} = 2{,}5$



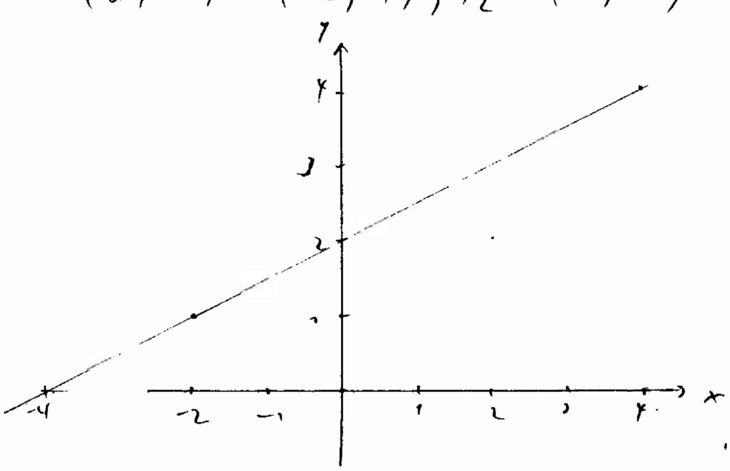
$m = -\frac{2}{3}$

$\alpha = -33{,}69°$

VK 4.2.

(oder $\frac{y-1}{x+2} = \frac{4-1}{4+2} = \frac{1}{2} \Rightarrow y = \frac{1}{2}(x+2) + 1$
$y = \frac{x}{2} + 2$)

(a) $P_1 = (-2; 1)$; $P_2 = (4; 4)$



$y = mx + b$

I. $1 = -2m + b$
II. $4 = 4m + b$

II−I $3 = 6m \Rightarrow m = \frac{1}{2}$

$b = 1 + 2m = 2$

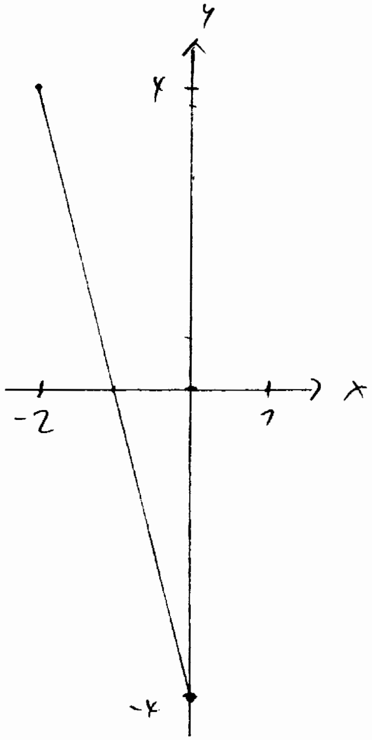
$\Rightarrow y = \frac{1}{2} x + 2$

$y = 0 \Leftrightarrow \frac{1}{2} x = -2 \Leftrightarrow x = -4$

zu VK 4.2.

$$\frac{y+4}{x-0} = \frac{-4+4}{-2-0} = -4 \Rightarrow y = -4x-4$$

(b) $P_1 = (0; -4)$; $P_2 = (-2; 4)$



$y = mx + b$

I: $-4 = b$

II: $4 = -2m - 4$

$\Rightarrow m = -\frac{8}{2} = -4$

$y = -4x - 4$

$y = 0 \Leftrightarrow x = -1$

VK 4.3.

(a) $y = -\frac{1}{4}x^2 - x + 3 = -\frac{1}{4}(x^2 + 4x - 12)$

$= -\frac{1}{4}((x+2)^2 - 16) = -\frac{1}{4}(x+2)^2 + 4$

$S = (-2; 4)$

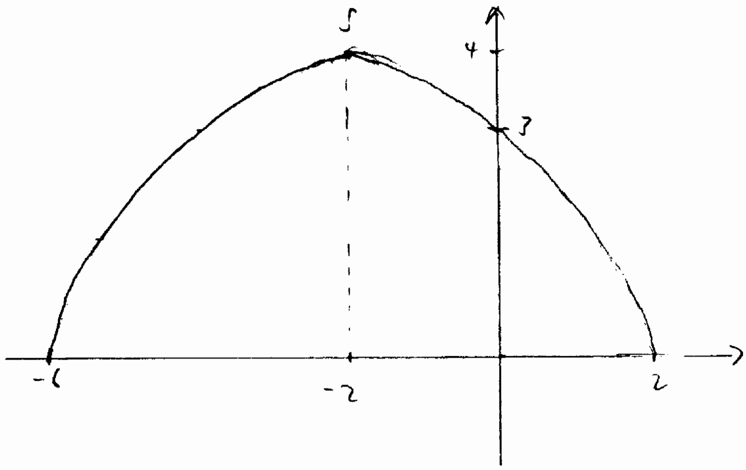
$y = 0 \Leftrightarrow (x+2)^2 - 16 = 0 \Leftrightarrow (x+2)^2 = 16$

$x_1 = 4 - 2 = 2$

$x_2 = -4 - 2 = -6$

$x = 0 \Rightarrow y = 3$

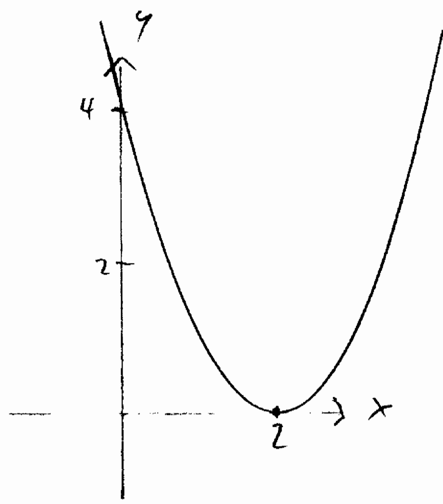
zu VK 4.3. (a)



$y = -\frac{1}{4}x^2 - x + 3$

(b) $y = (x-2)^2 \Rightarrow S = (2; 0)$; Nullstelle $x = 2$

$x = 0 \Rightarrow y = 4$

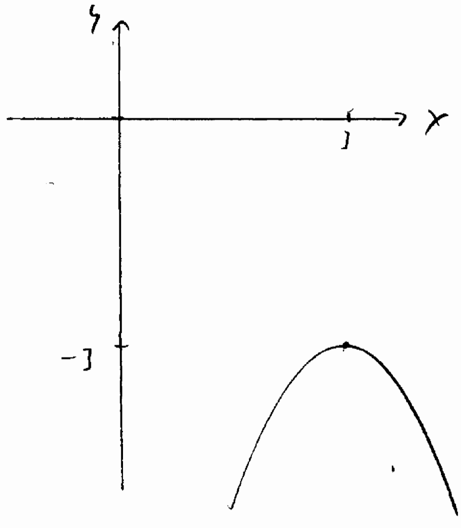


(c) $y = -(x^2 - 6x + 12) = -\left((x-3)^2 + 3\right) = -(x-3)^2 - 3$

$S = (3; -3)$; Nullstelle $y = 0 \Leftrightarrow (x-3)^2 = -3$

$\Leftrightarrow$ keine Nullstelle

$x = 0 \Rightarrow y = -12$



VK 4.4.

30

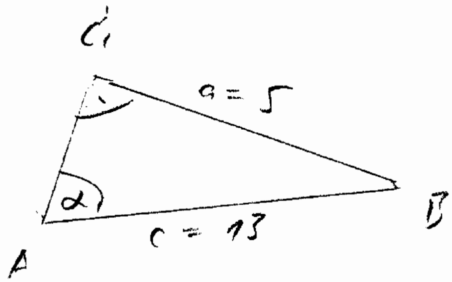
(a) $\sin(-390) = \sin(-30-360) = -\sin(30) = -\frac{1}{2}$

(b) $\cos\left(\frac{9\pi}{4}\right) = \cos\left(2\pi + \frac{\pi}{4}\right) = \cos\left(\frac{\pi}{4}\right) \triangleq \cos(45^\circ) = \frac{1}{\sqrt{2}}$

(c) $\arcsin\left(-\frac{1}{2}\sqrt{2}\right) = -\frac{\pi}{4} \triangleq -45^\circ = 315^\circ$

(d) $\arctan\left(\frac{1}{3}\sqrt{3}\right) = \arctan\left(\frac{1}{\sqrt{3}}\right) = 30^\circ$

VK 4.5



$\sin\alpha = \frac{5}{13}$; $\cos\alpha = \sqrt{1-\sin^2\alpha} = \sqrt{1-\frac{25}{169}} = \sqrt{\frac{144}{169}} = \frac{12}{13}$

$\tan\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{5}{13}\cdot\frac{13}{12} = \frac{5}{12} \Rightarrow \cot\alpha = \frac{12}{5}$

VK 4.6

$\cos\alpha = \frac{4}{5}$

$\sin\alpha = \sqrt{1-\cos^2\alpha} = \sqrt{1-\frac{16}{25}} = \sqrt{\frac{9}{25}} = \frac{3}{5}$

$\tan\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{3}{5}\cdot\frac{5}{4} = \frac{3}{4}$; $\cot\alpha = \frac{4}{3}$

VK 4.7

(a) $1+\cot^2\alpha = 1 + \frac{\cos^2\alpha}{\sin^2\alpha} = \frac{\sin^2\alpha}{\sin^2\alpha} + \frac{\cos^2\alpha}{\sin^2\alpha}$

$= \frac{\sin^2\alpha + \cos^2\alpha}{\sin^2\alpha} = \frac{1}{\sin^2\alpha}$

(b) $\cos\alpha = \cos\left(2\cdot\frac{\alpha}{2}\right) = \cos^2\left(\frac{\alpha}{2}\right) - \sin^2\left(\frac{\alpha}{2}\right)$

$= \cos^2\left(\frac{\alpha}{2}\right) - \left(1-\cos^2\left(\frac{\alpha}{2}\right)\right)$

$= \cos^2\left(\frac{\alpha}{2}\right) - 1 + \cos^2\left(\frac{\alpha}{2}\right) = 2\cos^2\left(\frac{\alpha}{2}\right) - 1$

$\Leftrightarrow \quad 2\cos^2\left(\frac{\alpha}{2}\right) = 1+\cos\alpha \quad | \cdot 2$

$\cos^2\left(\frac{\alpha}{2}\right) = \frac{1+\cos\alpha}{2} \quad | \sqrt{\ }$

$$\boxed{\cos\left(\frac{\alpha}{2}\right) = \sqrt{\frac{1+\cos(\alpha)}{2}}}$$

(c) $\cos(3\alpha) = \cos(2\alpha+\alpha) = \cos(2\alpha)\cos\alpha - \sin(2\alpha)\sin\alpha$

$= (\cos^2\alpha - \sin^2\alpha)\cos\alpha - 2\sin\alpha\cos\alpha\sin\alpha$

$= \cos^3\alpha - \sin^2\alpha\cos\alpha - 2\sin^2\alpha\cos\alpha$

$= \cos^3\alpha - 3\sin^2\alpha\cos\alpha$

$= \cos^3\alpha - 3(1-\cos^2\alpha)\cos\alpha$

$= \cos^3\alpha - 3\cos\alpha + 3\cos^3\alpha$

$= 4\cos^3\alpha - 3\cos\alpha$

(d) Wegen $\sin^2(3\pi-\alpha) + \cos^2(3\pi-\alpha) = 1$ genügt es zu zeigen

$\sin^2\left(\frac{17\pi}{2}+\alpha\right) \stackrel{!}{=} \cos^2(3\pi-\alpha)$

Es ist

$\sin\left(\frac{17\pi}{2}+\alpha\right) = \sin\left(6\pi+\frac{\pi}{2}+\alpha\right) = \sin\left(\frac{\pi}{2}+\alpha\right) = \cos(-\alpha)$

$= \cos(2\pi-\alpha) = -\cos(3\pi-\alpha) \Rightarrow$ Bel.

Beachte: $\cos(\pi+\beta) = -\cos\beta \ \forall \beta$ (Add. Theorem), $\cos\pi = -1$, $\sin\pi = 0$

VK4 8.

(a) $\sqrt{x+1-\sqrt{2x+3}} = 1 \iff x+1-\sqrt{2x+3} = 1$

$\iff x = \sqrt{2x+3} \implies x^2 = 2x+3$

$\iff x^2 - 2x - 3 = 0$

$x_{1/2} = \dfrac{2 \pm \sqrt{4+12}}{2} = \dfrac{2 \pm 4}{2} = 1 \pm 2$

$\implies x_1 = 3 \; ; \quad x_2 = -1$

Probe:

$x_1 = 3: \quad \sqrt{4-\sqrt{9}} = 1$ ✓

$x_2 = -1: \quad \sqrt{-\sqrt{1}} = \sqrt{-1} \neq 1$

$\implies \mathbb{L} = \{3\}$

(b) $\sqrt[3]{a-x} = b \iff a-x = b^3 \iff x = a-b^3$

(c) $\ln(x-1)^2 = 2 \iff (x-1)^2 = e^2 \implies x-1 = \pm e$

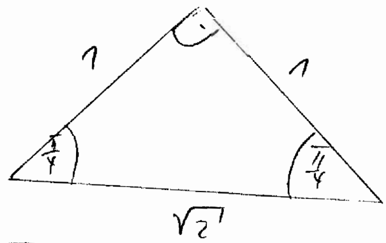
$\implies x_1 = 1+e \; ; \; x_2 = 1-e$

(d) $\left(\frac{6}{7}\right)^{3x+10} = \left(\frac{7}{6}\right)^{2x-3} \Leftrightarrow \left(\frac{6}{7}\right)^{3x+10} = \left(\frac{6}{7}\right)^{3-2x}$

$\Rightarrow 3x+10 = 3-2x \Leftrightarrow 5x = -7$

$\Leftrightarrow x = -\frac{7}{5}$

(e) $\sin x = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$



$\Rightarrow x_1 = \frac{\pi}{4}$

$x_2 = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$

beachte: $\sin x = \sin(\pi - x)$

$\mathbb{L} = \left\{\frac{\pi}{4} + 2k\pi \mid k \in \mathbb{Z}\right\} \cup \left\{\frac{3\pi}{4} + 2k\pi \mid k \in \mathbb{Z}\right\}$

(f) $2\cos^2 x + \sin x - 1 = 0$

$2(1-\sin^2 x) + \sin x - 1 = 0$

Substitution: $a := \sin x$

$2(1-a^2) + a - 1 = 0 \Leftrightarrow 2 - 2a^2 + a - 1 = 0$

$\Leftrightarrow 2a^2 - a - 1 = 0$

$a_{1/2} = \frac{1 \pm \sqrt{1+8}}{4} = \frac{1 \pm 3}{4}$

$\Rightarrow a_1 = 1 \;;\; a_2 = -\frac{2}{4} = -\frac{1}{2}$

1. Fall: $\sin x = 1 \Rightarrow x = \frac{\pi}{2} + 2k\pi \;;\; k \in \mathbb{Z}$

2. Fall: $\sin x = -\frac{1}{2} \Rightarrow x = \alpha + 2k\pi \;;\; k \in \mathbb{Z}$

mit $\alpha = -\frac{\pi}{6}$ oder $\alpha = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

$\Rightarrow \mathbb{L} = \left\{\frac{\pi}{2} + 2k\pi \mid k \in \mathbb{Z}\right\} \cup \left\{-\frac{\pi}{6} + 2k\pi \mid k \in \mathbb{Z}\right\}$

$\cup \left\{\frac{5\pi}{6} + 2k\pi \mid k \in \mathbb{Z}\right\}$