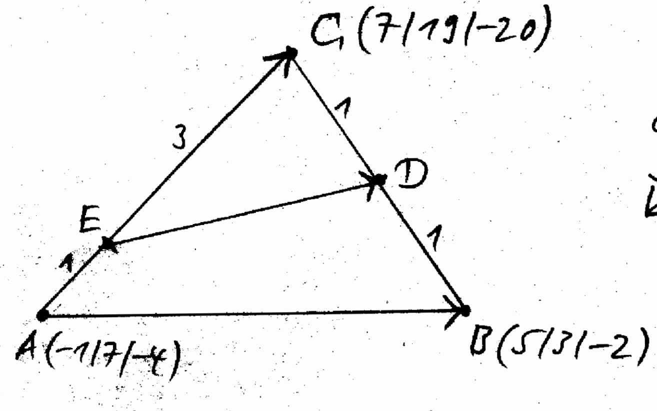
Lösungen zur Vektorrechnung:

Afg 1



$\vec{a} = \overrightarrow{AB}$

$\vec{b} = \overrightarrow{AC}$

a) Koordinaten von $\vec{a}$ und $\vec{b}$:

$$\vec{a} = \overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \begin{pmatrix} 5 \\ 3 \\ -2 \end{pmatrix} - \begin{pmatrix} -1 \\ 7 \\ -4 \end{pmatrix} = \begin{pmatrix} 5+1 \\ 3-7 \\ -2+4 \end{pmatrix} = \begin{pmatrix} 6 \\ -4 \\ 2 \end{pmatrix}$$

$$\vec{b} = \overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA} = \begin{pmatrix} 7 \\ 19 \\ -20 \end{pmatrix} - \begin{pmatrix} -1 \\ 7 \\ -4 \end{pmatrix} = \begin{pmatrix} 8 \\ 12 \\ -16 \end{pmatrix}$$

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b) Koordinaten des Vektors $\overrightarrow{ED}$:

Stelle $\overrightarrow{ED}$ zunächst als Linearkombination von $\vec{a}$ und $\vec{b}$ dar und berechne sodann die Koordinaten!

$$\overrightarrow{ED} = \overrightarrow{EC} + \overrightarrow{CD}$$
$$= \tfrac{3}{4}\overrightarrow{AC} + \tfrac{1}{2}\overrightarrow{CB} = \tfrac{3}{4}\vec{b} + \tfrac{1}{2}(\vec{a} - \vec{b})$$
$$= \tfrac{3}{4}\vec{b} + \tfrac{1}{2}\vec{a} - \tfrac{1}{2}\vec{b} = \tfrac{1}{2}\vec{a} + \tfrac{1}{4}\vec{b}$$
$$= \tfrac{1}{2}\begin{pmatrix} 6 \\ -4 \\ 2 \end{pmatrix} + \tfrac{1}{4}\begin{pmatrix} 8 \\ 12 \\ -16 \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix} + \begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix} = \begin{pmatrix} 5 \\ 1 \\ -3 \end{pmatrix}$$

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c) Koordinaten von E und D:

$$\overrightarrow{AE} = \tfrac{1}{4}\vec{b} \iff \overrightarrow{OE} - \overrightarrow{OA} = \tfrac{1}{4}\vec{b}$$

$$\Rightarrow \overrightarrow{OE} = \tfrac{1}{4}\vec{b} + \overrightarrow{OA} = \tfrac{1}{4}\begin{pmatrix} 8 \\ 12 \\ -16 \end{pmatrix} + \begin{pmatrix} -1 \\ 7 \\ -4 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix} + \begin{pmatrix} -1 \\ 7 \\ -4 \end{pmatrix} = \begin{pmatrix} 1 \\ 10 \\ -8 \end{pmatrix}$$

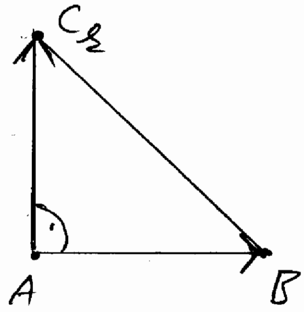
$$\Rightarrow \boxed{E(1/10/-8)}$$

$$\overrightarrow{OD} - \overrightarrow{OE} = \overrightarrow{ED} = \begin{pmatrix} 5 \\ 1 \\ -3 \end{pmatrix} \Rightarrow \overrightarrow{OD} = \begin{pmatrix} 5 \\ 1 \\ -3 \end{pmatrix} + \overrightarrow{OE} = \begin{pmatrix} 5 \\ 1 \\ -3 \end{pmatrix} + \begin{pmatrix} 1 \\ 10 \\ -8 \end{pmatrix} = \begin{pmatrix} 6 \\ 11 \\ -11 \end{pmatrix}$$

$$\Rightarrow \boxed{D(6/11/-11)}$$

Afg 2 $\quad A(-2|3|4),\ B(5|-1|0),\ C_k(3|k|-2),\ k\in\mathbb{R}$ $\quad$ ②

a)



k so berechnen, dass das $\Delta\, ABC_k$ bei A rechtwinklig ist!

Es gilt: $\quad \vec{AB} \perp \vec{AC_k} \iff \vec{AB}\circ\vec{AC_k} = 0$

Zunächst Vektoren berechnen:

$$\vec{AB} = \vec{OB} - \vec{OA} = \begin{pmatrix}5\\-1\\0\end{pmatrix} - \begin{pmatrix}-2\\3\\4\end{pmatrix} = \begin{pmatrix}7\\-4\\-4\end{pmatrix}$$

$$\vec{AC_k} = \vec{OC_k} - \vec{OA} = \begin{pmatrix}3\\k\\-2\end{pmatrix} - \begin{pmatrix}-2\\3\\4\end{pmatrix} = \begin{pmatrix}5\\k-3\\-6\end{pmatrix}$$

$$0 = \vec{AB}\circ\vec{AC_k} = \begin{pmatrix}7\\-4\\-4\end{pmatrix}\circ\begin{pmatrix}5\\k-3\\-6\end{pmatrix} = 7\cdot 5 - 4(k-3) - 4\cdot(-6)$$
$$= 35 - 4(k-3) + 24$$

$$\iff 4(k-3) = 59 \iff \boxed{k = 17{,}75}$$

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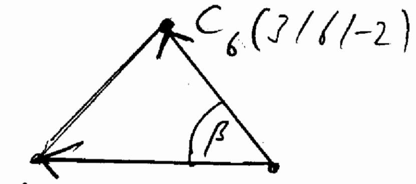
b) Sei $k = 6$. Winkel β des Dreiecks ABC_6 berechnen:



$$\cos\beta = \frac{\vec{BA}\circ\vec{BC_6}}{|\vec{BA}|\cdot|\vec{BC_6}|}$$

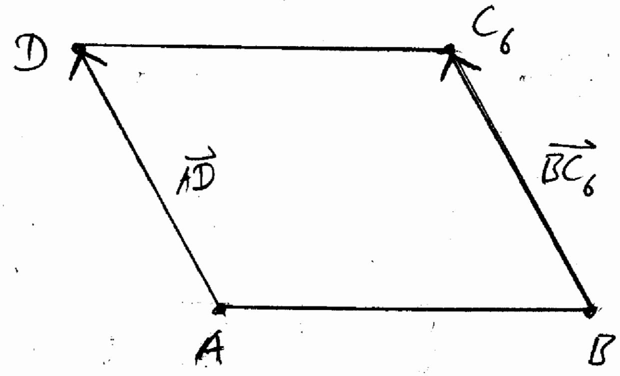
$$\vec{BA} = -\vec{AB} = \begin{pmatrix}-7\\4\\4\end{pmatrix};\quad \vec{BC_6} = \begin{pmatrix}3\\6\\-2\end{pmatrix} - \begin{pmatrix}5\\-1\\0\end{pmatrix} = \begin{pmatrix}-2\\7\\-2\end{pmatrix}$$

$$\vec{BA}\circ\vec{BC_6} = \begin{pmatrix}-7\\4\\4\end{pmatrix}\circ\begin{pmatrix}-2\\7\\-2\end{pmatrix} = 14 + 28 - 8 = 34$$

$$|\vec{BA}| = \sqrt{(-7)^2+4^2+4^2} = 9;\quad |\vec{BC_6}| = \sqrt{(-2)^2+7^2+(-2)^2} = \sqrt{57}$$

$$\Rightarrow \beta = \cos^{-1}\left(\frac{34}{9\sqrt{57}}\right) = \boxed{60°}$$

zu Afg 2c



Koordinaten von D so berechnen, dass das Viereck ABC_6D zu einem Parallelogramm wird!

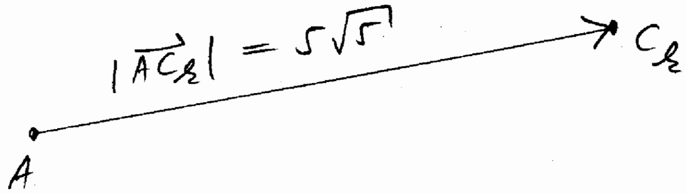
Es muss gelten: $\overrightarrow{AD} = \overrightarrow{BC_6} \iff \overrightarrow{OD} - \overrightarrow{OA} = \overrightarrow{BC_6}$

$$\Rightarrow \overrightarrow{OD} = \overrightarrow{BC_6} + \overrightarrow{OA} = \begin{pmatrix} -2 \\ 7 \\ -2 \end{pmatrix} + \begin{pmatrix} -2 \\ 3 \\ 4 \end{pmatrix} = \begin{pmatrix} -4 \\ 10 \\ 2 \end{pmatrix}$$

$$\Rightarrow \boxed{D(-4|10|2)}$$

Afg 3 $A(-6|0|2)$; $B(4|-10|8)$, $C_k(k|k+3|-4)$, $k \in \mathbb{R}$

a)



$k = ?$

$$|\overrightarrow{AC_k}| = |\overrightarrow{OC_k} - \overrightarrow{OA}| = \left| \begin{pmatrix} k+6 \\ k+3 \\ -6 \end{pmatrix} \right| = \sqrt{(k+6)^2 + (k+3)^2 + 36}$$

Löse die Gleichung $\sqrt{(k+6)^2 + (k+3)^2 + 36} = 5\sqrt{5}$ nach k auf!

Quadrieren ergibt

$$(k+6)^2 + (k+3)^2 + 36 = 125$$

$$k^2 + 12k + 36 + k^2 + 6k + 9 + 36 = 125$$

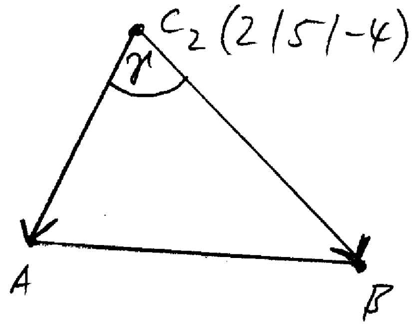
$$2k^2 + 18k + 81 = 125 \iff 2k^2 + 18k - 44 = 0 \quad |:2$$

$$\iff k^2 + 9k - 22 = 0$$

$$k_{1/2} = \frac{-9 \pm \sqrt{81 + 4 \cdot 22}}{2} = \frac{-9 \pm 13}{2} \Rightarrow \boxed{k_1 = 2}; \boxed{k_2 = -11}$$

$$\Rightarrow \boxed{C_2(2|5|-4)}; \boxed{C_{-11}(-11|-8|-4)}$$

3b) Sei $k = 2$. γ berechnen!



$$\cos\gamma = \frac{\overrightarrow{C_2A} \circ \overrightarrow{C_2B}}{|\overrightarrow{C_2A}| \cdot |\overrightarrow{C_2B}|}$$

$$\overrightarrow{C_2A} = \overrightarrow{OA} - \overrightarrow{OC_2} = \begin{pmatrix}-6\\0\\2\end{pmatrix} - \begin{pmatrix}2\\5\\-4\end{pmatrix} = \begin{pmatrix}-8\\-5\\6\end{pmatrix}$$

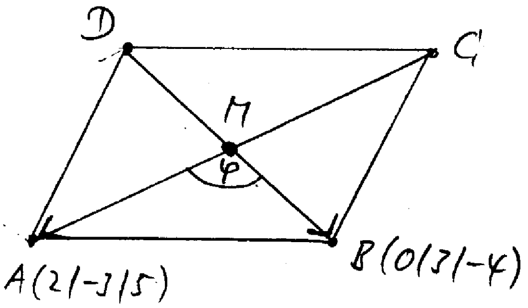
$$\overrightarrow{C_2B} = \overrightarrow{OB} - \overrightarrow{OC_2} = \begin{pmatrix}4\\-10\\8\end{pmatrix} - \begin{pmatrix}2\\5\\-4\end{pmatrix} = \begin{pmatrix}2\\-15\\12\end{pmatrix}$$

$$\overrightarrow{C_2A} \circ \overrightarrow{C_2B} = \begin{pmatrix}-8\\-5\\6\end{pmatrix} \cdot \begin{pmatrix}2\\-15\\12\end{pmatrix} = -16 + 75 + 72 = 131$$

$$|\overrightarrow{C_2A}| = \sqrt{64+25+36} \overset{a)}{=} 5\sqrt{5}\,; \quad |\overrightarrow{C_2B}| = \sqrt{4+15^2+12^2} = \sqrt{373}$$

$$\gamma = \cos^{-1}\left(\frac{131}{5\sqrt{5}\sqrt{373}}\right) = \boxed{52{,}65°}$$

Afg 4



$M(4|3|-1)$

a) C und D berechnen:

$\overrightarrow{AC} = 2\overrightarrow{AM} \Leftrightarrow \overrightarrow{OC} - \overrightarrow{OA} = 2\overrightarrow{AM}$

$\overrightarrow{OC} = \overrightarrow{OA} + 2\overrightarrow{AM} = \begin{pmatrix}2\\-3\\5\end{pmatrix} + 2\begin{pmatrix}2\\6\\-6\end{pmatrix}$

$\overrightarrow{OC} = \begin{pmatrix}6\\9\\-7\end{pmatrix} \Rightarrow \boxed{C(6|9|-7)}$

$\overrightarrow{AD} = \overrightarrow{BC} \Leftrightarrow \overrightarrow{OD} - \overrightarrow{OA} = \overrightarrow{BC}$

$\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{BC} = \begin{pmatrix}2\\-3\\5\end{pmatrix} + \begin{pmatrix}6\\6\\-3\end{pmatrix}$

$\overrightarrow{OD} = \begin{pmatrix}8\\3\\2\end{pmatrix} \Rightarrow \boxed{D(8|3|2)}$

b) Umfang des Parallelogramms

$U = 2\,(|\overrightarrow{AB}| + |\overrightarrow{BC}|)$

$= 2\left(\left|\begin{pmatrix}-2\\6\\-9\end{pmatrix}\right| + \left|\begin{pmatrix}6\\6\\-3\end{pmatrix}\right|\right)$

$= 2\left(\sqrt{4+36+81} + \sqrt{36+36+9}\right)$

$= 2\,(11 + 9) = 2 \cdot 20$

$= \boxed{40 \text{ LE}}$

c) $\varphi = \sphericalangle AMB$?

$\cos\varphi = \dfrac{\overrightarrow{MA} \circ \overrightarrow{MB}}{|\overrightarrow{MA}| \cdot |\overrightarrow{MB}|}$

$\overrightarrow{MA} = \begin{pmatrix}-2\\-6\\6\end{pmatrix}; \quad \overrightarrow{MB} = \begin{pmatrix}-4\\0\\-3\end{pmatrix}$

$\overrightarrow{MA} \circ \overrightarrow{MB} = 8 - 18 = -10$

$|\overrightarrow{MA}| = \sqrt{4+36+36} = 2\sqrt{19}$

$|\overrightarrow{MB}| = \sqrt{16+9} = 5$

$\varphi = \cos^{-1}\left(\dfrac{-10}{10\sqrt{19}}\right) = \boxed{103{,}26°}$