Aufgabe 3

a) $\int (4x^3-5x^2+3x+7)\,dx$

$$= 4\int x^3dx - 5\int x^2 dx + 3\int x\,dx + 7\int 1\,dx$$

$$= 4\cdot\frac{x^4}{4} - 5\cdot\frac{x^3}{3} + 3\cdot\frac{x^2}{2} + 7x$$

$$= \boxed{x^4 - \frac{5x^3}{3} + \frac{3x^2}{2} + 7x}$$

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b) zunächst Stammfunktion $F(x)$ berechnen:

$$F(x) = \int\left(\frac{2x^3}{5} - \frac{x}{2}\right)dx = \frac{2}{5}\int x^3dx - \frac{1}{2}\int x\,dx$$

$$= \frac{2}{5}\cdot\frac{x^4}{4} - \frac{1}{2}\cdot\frac{x^2}{2} = \frac{x^4}{10} - \frac{x^2}{4}$$

daraus

$$\int_{-\sqrt{3}}^{\sqrt{2}}\left(\frac{2x^3}{5} - \frac{x}{2}\right)dx = \left[\frac{x^4}{10} - \frac{x^2}{4}\right]_{-\sqrt{3}}^{\sqrt{2}}$$

$$= F(\sqrt{2}) - F(-\sqrt{3})$$

$$= \left(\frac{(\sqrt{2})^4}{10} - \frac{(\sqrt{2})^2}{4}\right) - \left(\frac{(-\sqrt{3})^4}{10} - \frac{(-\sqrt{3})^2}{4}\right)$$

$$= \left(\frac{4}{10} - \frac{2}{4}\right) - \left(\frac{9}{10} - \frac{3}{4}\right)$$

$$= \frac{2}{5} - \frac{1}{2} - \frac{9}{10} + \frac{3}{4} = \frac{8-10-18+15}{20} = -\frac{5}{20}$$

$$= \boxed{-\frac{1}{4}}$$

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c) $\int\left(2 - \frac{3x^2}{4} - 4x\right)dx = \int 2\,dx - \frac{3}{4}\int x^2dx - 4\int x\,dx$

$$= 2x - \frac{\cancel{3}}{4}\cdot\frac{x^3}{\cancel{3}} - 4\cdot\frac{x^2}{2}$$

$$= \boxed{2x - \frac{x^3}{4} - 2x^2}$$

②

d) zunächst Stammfunktion $F(x)$ berechnen:

$$F(x) = \int \left(3\cos x - \frac{2}{x}\right)dx = 3\int \cos x\, dx - 2\int \frac{1}{x}dx$$
$$= 3\sin x - 2\ln|x|$$

daraus

$$\int_{-\sqrt{e}}^{\frac{\pi}{2}} \left(3\cos x - \frac{2}{x}\right)dx = \Big[3\sin x - 2\ln|x|\Big]_{-\sqrt{e}}^{\frac{\pi}{2}} = F\left(\tfrac{\pi}{2}\right) - F(-\sqrt{e})$$
$$= \left(3\sin\frac{\pi}{2} - 2\ln\frac{\pi}{2}\right) - \left(3\sin(-\sqrt{e}) - 2\ln|-\sqrt{e}|\right)$$
$$= \left(3 - 2\ln\frac{\pi}{2}\right) - \left(-3\sin\sqrt{e} - 2\ln\sqrt{e}\right)$$
$$= 3 - 2\ln\frac{\pi}{2} + 3\sin\sqrt{e} + 2\cdot\frac{1}{2}\underbrace{\ln e}_{=1}$$
$$= 5 - 2\ln\frac{\pi}{2} + 3\sin\sqrt{e}$$
$$\approx \boxed{7{,}0877}$$

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e) zunächst Stammfunktion $F(x)$ berechnen:

$$F(x) = \int\left(\frac{e^x}{3} - \frac{\sin x}{4}\right)dx$$
$$= \frac{1}{3}\int e^x\,dx - \frac{1}{4}\int \sin x\, dx$$
$$= \frac{e^x}{3} + \frac{\cos x}{4}$$

daraus

$$\int_0^{2\pi}\left(\frac{e^x}{3} - \frac{\sin x}{4}\right)dx = \left[\frac{e^x}{3} + \frac{\cos x}{4}\right]_0^{2\pi}$$
$$= F(2\pi) - F(0)$$
$$= \left(\frac{e^{2\pi}}{3} + \frac{\cos 2\pi}{4}\right) - \left(\frac{e^0}{3} + \frac{\cos 0}{4}\right)$$
$$= \left(\frac{e^{2\pi}}{3} + \frac{1}{4}\right) - \left(\frac{1}{3} + \frac{1}{4}\right)$$
$$= \frac{e^{2\pi}}{3} + \cancel{\frac{1}{4}} - \frac{1}{3} - \cancel{\frac{1}{4}} = \boxed{\frac{e^{2\pi}-1}{3} \approx 178{,}1639}$$

f) $\int \left(\frac{\sin x}{2} + \frac{3}{x^2} \right) dx = \frac{1}{2} \int \sin x \, dx + 3 \int x^{-2} dx$

$= -\frac{1}{2} \cos x + 3 \cdot \frac{x^{-1}}{-1} = \boxed{-\frac{1}{2} \cos x - \frac{3}{x}}$

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g) zunächst Stammfunktion $F(x)$ berechnen:

$F(x) = \int \left(\frac{\sqrt[3]{x}}{4} + \frac{2}{\sqrt{x}} \right) dx$

$= \frac{1}{4} \int x^{\frac{1}{3}} dx + 2 \int x^{-\frac{1}{2}} dx$

$= \frac{1}{4} \cdot \frac{x^{4/3}}{4/3} + 2 \cdot \frac{x^{1/2}}{1/2}$

$= \frac{1}{4} \cdot \frac{3}{4} \cdot x^{4/3} + 2 \cdot 2 \cdot x^{1/2}$

$F(x) = \boxed{\frac{3}{16} \cdot x^{4/3} + 4 \cdot x^{\frac{1}{2}}} \left[= \frac{3 \cdot \sqrt[3]{x^4}}{16} + 4\sqrt{x} \right]$

daraus:

$\int_{e^3}^{e^2} \left(\frac{\sqrt[3]{x}}{4} + \frac{2}{\sqrt{x}} \right) dx = \left[\frac{3}{16} \cdot x^{\frac{4}{3}} + 4 \cdot x^{\frac{1}{2}} \right]_{e^3}^{e^2}$

$= F(e^2) - F(e^3)$

$= \left(\frac{3}{16} \cdot (e^2)^{\frac{4}{3}} + 4 \cdot (e^2)^{\frac{1}{2}} \right) - \left(\frac{3}{16} \cdot (e^3)^{\frac{4}{3}} + 4 \cdot (e^3)^{\frac{1}{2}} \right)$

$= \left(\frac{3}{16} \cdot e^{\frac{8}{3}} + 4e \right) - \left(\frac{3}{16} \cdot e^4 + 4 \cdot e^{\frac{3}{2}} \right)$

$= 13{,}57 - 28{,}16 = \boxed{-14{,}59}$

h) zunächst Stammfunktion $F(x)$ berechnen:

$$F(x) = \int \left(1 - \frac{5x}{3}\right)^2 dx$$

$$= \frac{\left(1 - \frac{5x}{3}\right)^3}{3} \cdot \frac{1}{-5/3}$$

$$= -\frac{1}{3} \cdot \frac{3}{5}\left(1 - \frac{5x}{3}\right)^3 = -\frac{1}{5}\left(1 - \frac{5x}{3}\right)^3$$

daraus:

$$\int_{\sqrt{\pi - e}}^{\pi + e} \left(1 - \frac{5x}{3}\right)^2 dx = \left[-\frac{1}{5}\left(1 - \frac{5x}{3}\right)^3\right]_{\sqrt{\pi - e}}^{\pi + e}$$

$$= F(\pi + e) - F(\sqrt{\pi - e})$$

$$= \left(-\frac{1}{5}\left(1 - \frac{5(\pi + e)}{3}\right)^3\right) - \left(-\frac{1}{5}\left(1 - \frac{5\sqrt{\pi - e}}{3}\right)^3\right)$$

$$= 134{,}74 - 0{,}00012 = \boxed{134{,}73988}$$

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i) zunächst Stammfunktion $F(x)$ berechnen:

$$F(x) = \int \frac{5}{6x+13}\, dx = 5 \int (6x+13)^{-1}\, dx$$

$$= 5 \ln|6x+13| \cdot \frac{1}{6} = \frac{5}{6} \ln|6x+13|$$

daraus:

$$\int_{1}^{-3} \frac{5}{6x+13}\, dx = \left[\frac{5}{6} \ln|6x+13|\right]_{1}^{-3}$$

$$= \frac{5}{6} \ln|6 \cdot (-3) + 13| - \frac{5}{6} \ln|6 \cdot 1 + 13|$$

$$= \frac{5}{6} \ln|-5| - \frac{5}{6} \ln|19|$$

$$= \frac{5}{6}(\ln 5 - \ln 19) = \boxed{\frac{5}{6} \ln\left(\frac{5}{19}\right)}$$

$$\approx \boxed{-1{,}1125}$$

j) $\int \frac{2}{(\frac{x}{4}+2)^3}\,dx = 2\int (\frac{x}{4}+2)^{-3}\,dx$

$= \cancel{2}\,\frac{(\frac{x}{4}+2)^{-2}}{-\cancel{2}} \cdot \frac{1}{1/4}$

$= -4\,(\frac{x}{4}+2)^{-2} = \boxed{-\frac{4}{(\frac{x}{4}+2)^2}}$

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k) zunächst Stammfunktion $F(x)$ berechnen:

$F(x) = \int 10\sin(4-\frac{x}{5})\,dx = 10\int \sin(4-\frac{x}{5})\,dx$

$= -10\cos(4-\frac{x}{5}) \cdot \frac{1}{-1/5}$

$= 50\cos(4-\frac{x}{5})$

daraus:

$\int_{5(4-\pi)}^{0} 10\sin(4-\frac{x}{5})\,dx = \left[50\cos(4-\frac{x}{5})\right]_{5(4-\pi)}^{0}$

$= F(0) - F(5(4-\pi))$

$= 50\cos(4-\frac{0}{5}) - 50\cos(4-\frac{5(4-\pi)}{5})$

$= 50\cos 4 - 50\cos(4-4+\pi)$

$= 50(\cos(4) - \underbrace{\cos\pi}_{=-1})$

$= \boxed{50(\cos(4)+1)} \approx \boxed{17{,}32}$

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l) $\int \frac{\sqrt{2}}{e^{2x}}\,dx = \sqrt{2}\int e^{-2x}\,dx$

$= \sqrt{2}\cdot e^{-2x}\cdot\frac{1}{-2} = \boxed{-\frac{1}{\sqrt{2}\cdot e^{2x}}}$

m) zunächst Stammfunktion $F(x)$ berechnen:

$$F(x)=\int \frac{\sqrt{e^x}}{2}\,dx = \frac{1}{2}\int e^{\frac{x}{2}}\,dx$$

$$= \frac{1}{2}\cdot e^{\frac{x}{2}}\cdot \frac{1}{1/2} = \sqrt{e^x}$$

daraus:

$$\int_{\ln 4}^{\ln 9} \frac{\sqrt{e^x}}{2}\,dx = \left[\sqrt{e^x}\right]_{\ln 4}^{\ln 9} = F(\ln 9)-F(\ln 4)$$

$$= \sqrt{e^{\ln 9}} - \sqrt{e^{\ln 4}}$$

$$= \sqrt{9}-\sqrt{4} = 3-2 = \boxed{1}$$

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n) zunächst Stammfunktion $F(x)$ berechnen:

$$F(x) = \int\sqrt{4x-7}\,dx = \int (4x-7)^{1/2}dx = \frac{1}{4}\cdot\frac{(4x-7)^{3/2}}{3/2}$$

$$= \frac{1}{4}\cdot\frac{2}{3}\cdot(4x-7)^{3/2} = \frac{1}{6}(4x-7)^{3/2}$$

daraus $\displaystyle\int_2^3 \sqrt{4x-7}\,dx = \frac{1}{6}(4\cdot 3-7)^{3/2} - \frac{1}{6}(4\cdot 2-7)^{3/2}$

$$= \boxed{\frac{1}{6}\left(5^{3/2}-1\right)} \approx \boxed{1{,}6967}$$

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o) $(3x^2-2x+1):(x-1) = 3x+1+\dfrac{2}{x-1}$

$-(3x^2-3x)$

$\quad x+1$

$\quad -(x-1)$

$\quad\quad 2$

daraus

$$\int\frac{3x^2-2x+1}{x-1}\,dx = \int\left(3x+1+\frac{2}{x-1}\right)dx$$

$$= 3\int x\,dx + \int 1\,dx + 2\int\frac{1}{x-1}\,dx$$

$$= \boxed{\frac{3x^2}{2} + x + 2\ln|x-1|}$$

Aufgabe 4

Flächeninhalt zwischen x-Achse und Funktionsgraph bestimmen!

a) $f(x) = \frac{1}{2}(x^2 - 2x - 8)$

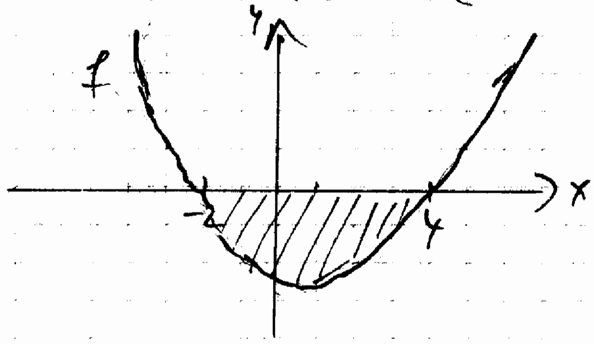
zunächst Nullstellen von f bestimmen:

$f(x) = 0 \iff x^2 - 2x - 8 = 0$

$$x_{1,2} = \frac{2 \pm \sqrt{4+32}}{2} = \frac{2 \pm 6}{2} = 1 \pm 3$$

$\boxed{x_1 = 4}$; $\boxed{x_2 = -2}$

Jetzt von Nullstelle zu Nullstelle integrieren:



$$F(x) = \int \frac{1}{2}(x^2 - 2x - 8)\,dx = \frac{1}{2}\int (x^2 - 2x - 8)\,dx = \frac{1}{2}\left(\frac{x^3}{3} - x^2 - 8x\right)$$

$$\text{Fläche} = \left|\int_{-2}^{4} f(x)\,dx\right| = |F(4) - F(-2)| = \left|-\frac{40}{3} - \frac{14}{3}\right| = \boxed{18 \text{ FE}}$$

b) $g(x) = \frac{1}{10}(x-1)(x^2-16)$

$$= \frac{1}{10}(x^3 - 16x - x^2 + 16)$$

$$= \frac{1}{10}(x^3 - x^2 - 16x + 16)$$

zunächst Nullstellen von g bestimmen:

sieht man an der gegebenen Form:

$x - 1 = 0 \quad \vee \quad x^2 - 16 = 0$

$\boxed{x_1 = 1}$ $\quad \boxed{x_2 = 4}$; $\boxed{x_3 = -4}$

$$G(x) = \frac{1}{10}\int (x^3 - x^2 - 16x + 16)\,dx$$

$$= \frac{1}{10}\left(\frac{x^4}{4} - \frac{x^3}{3} - 8x^2 + 16x\right)$$

$$G(1) = \frac{19}{24} \;;\quad G(4) = -\frac{32}{15} \;;\quad G(-4) = -\frac{32}{3}$$

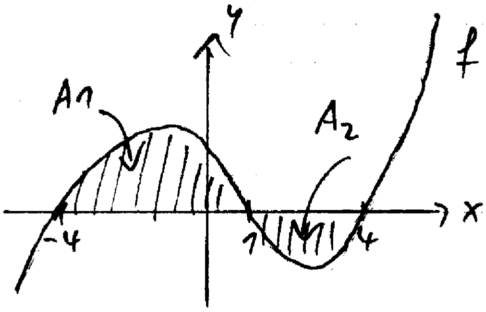
$$A_1 = \int_{-4}^{1} g(x)\,dx = G(1) - G(-4)$$

$$= \frac{19}{24} - \left(-\frac{32}{3}\right) = \frac{19}{24} + \frac{32}{3} = \frac{275}{24}$$

$$A_2 = \left|\int_{1}^{4} g(x)\,dx\right| = |G(4) - G(1)|$$

$$= \left|-\frac{32}{15} - \frac{19}{24}\right| = \frac{117}{40}$$

Gesamtfläche $= A_1 + A_2 = \frac{863}{60} = \boxed{14{,}38\overline{3}\ \text{FE}}$



Aufgabe 5

a) $s(t) = \int v(t)\,dt = -7 \cdot \frac{\left(\frac{t}{5}-1\right)^5}{5} \cdot \frac{1}{1/5} + 7t + C$

$$s(t) = -7\left(\frac{t}{5}-1\right)^5 + 7t + C$$

Anfangsbed: $s(0) = 0 \iff 0 = -7(-1)^5 + C$

$$\iff 7 + C = 0 \iff C = -7$$

$\Rightarrow \boxed{s(t) = -7\left(\frac{t}{5}-1\right)^5 + 7t - 7}$ Weg-Zeit-Funktion

b) $\int_{1}^{2} v(t)\,dt = s(2) - s(1)$

$$= 7{,}544 - 2{,}294 = \boxed{5{,}25\ \text{m}}$$