Lösungen zu Übung 6

Afg 1

1) $$y = (2x^3+5x-4)(5x^2-3)$$

$$y' = (2x^3+5x-4)'(5x^2-3) + (2x^3+5x-4)(5x^2-3)'$$

$$= (6x^2+5)(5x^2-3) + 10x(2x^3+5x-4)$$

$$= 30x^4 - 18x^2 + 25x^2 - 15 + 20x^4 + 50x^2 - 40x$$

$$= 50x^4 + 57x^2 - 40x - 15$$

---

2) $$y = (\sin x + 3x^2)\cos x + 3x^4 - 1$$

$$y' = (\sin x + 3x^2)'\cos x + (\sin x + 3x^2)\cos' x + 12x^3$$

$$= (\cos x + 6x)\cos x - \sin x(\sin x + 3x^2) + 12x^3$$

$$\left[= \cos^2 x + 6x\cos x - \sin^2 x - 3x^2 \sin x + 12x^3\right]$$

---

3) $$y = (\sin^2 x)\sqrt{x}$$

$$y' = (\sin^2 x)'\sqrt{x} + (\sin^2 x)(\sqrt{x})'$$

$$= 2\sin x\cos x\sqrt{x} + \sin^2 x \cdot \frac{1}{2\sqrt{x}}$$

$$= \sqrt{x}\left(2\sin x\cos x + \frac{\sin^2 x}{2x}\right)$$

Afg 2

1) $y = \frac{x}{x-1}$

$$y' = \frac{x'(x-1) - x(x-1)'}{(x-1)^2} = \frac{x-1-x}{(x-1)^2} = -\frac{1}{(x-1)^2}$$

---

2) $y = \frac{x^5 - 2x}{3x-1}$

$$y' = \frac{(x^5-2x)'(3x-1) - (x^5-2x)(3x-1)'}{(3x-1)^2}$$

$$= \frac{(5x^4-2)(3x-1) - 3(x^5-2x)}{(3x-1)^2}$$

$$= \frac{15x^5 - 5x^4 \cancel{-6x} + 2 - 3x^5 \cancel{+6x}}{(3x-1)^2} = \frac{12x^5 - 5x^4 + 2}{(3x-1)^2}$$

---

3) $y = \frac{\sqrt{x} - \cos x}{3e^x + x^2}$

$$y' = \frac{(\sqrt{x} - \cos x)'(3e^x + x^2) - (\sqrt{x} - \cos x)(3e^x + x^2)'}{(3e^x+x^2)^2}$$

$$= \frac{\left(\frac{1}{2\sqrt{x}} + \sin x\right)(3e^x + x^2) - (\sqrt{x} - \cos x)(3e^x + 2x)}{(3e^x + x^2)^2}$$

$$\left[\underset{\text{erweitern}}{\overset{\text{mit } 2\sqrt{x}}{=}} \frac{(1 + 2\sqrt{x}\sin x)(3e^x + x^2) - (2x - 2\sqrt{x}\cos x)(3e^x + 2x)}{2(3e^x+x^2)^2\sqrt{x}} \right]$$

---

4) $y = \cot x = \frac{\cos x}{\sin x}$

$$y' = \frac{\cos' x \sin x - \cos x \sin' x}{\sin^2 x} = \frac{-\sin^2 x - \cos^2 x}{\sin^2 x}$$

$$= -\frac{1}{\sin^2 x} \text{ oder } -1 - \cot^2 x$$

Afg 3

1) $y = \sqrt{x^2+3} - 5(4x-1)^3$

$= (x^2+3)^{\frac{1}{2}} - 5(4x-1)^3$

$y' = \frac{1}{2}(x^2+3)^{-\frac{1}{2}} \cdot 2x - 15(4x-1)^2 \cdot 4$

$= \frac{x}{\sqrt{x^2+3}} - 60(4x-1)^2$

---

2) $y = \frac{1}{x^3-1} = (x^3-1)^{-1}$

$y' = -(x^3-1)^{-2} \cdot 3x^2 = -\frac{3x^2}{(x^3-1)^2}$

---

3) $y = \frac{1}{\sqrt{4-x^2}} = (4-x^2)^{-\frac{1}{2}}$

$y' = -\frac{1}{2}(4-x^2)^{-\frac{3}{2}} \cdot (-2x) = \frac{x}{\sqrt{(4-x^2)^3}}$

---

4) $y = e^{\sqrt{x}} \Rightarrow y' = e^{\sqrt{x}} \cdot (\sqrt{x})' = \frac{e^{\sqrt{x}}}{2\sqrt{x}}$

---

5) $y = \ln(3x^2+2x-1)$

$y' = \frac{(3x^2+2x-1)'}{3x^2+2x-1} = \frac{6x+2}{3x^2+2x-1}$

zu Afg 3

6) $y = \cos\left(\sqrt{x^3+\sin^2 x}\right) = \cos\left((x^3+\sin^2 x)^{\frac{1}{2}}\right)$

$$y' = -\sin\left(\sqrt{x^3+\sin^2 x}\right)\cdot \frac{1}{2}(x^3+\sin^2 x)^{-\frac{1}{2}}(3x^2+2\sin x\cos x)$$

$$= -\frac{3x^2+2\sin x\cos x}{2\sqrt{x^3+\sin^2 x}}\sin\left(\sqrt{x^3+\sin^2 x}\right)$$

$\boxed{\text{Schreibweise } \exp(z)=e^z}$

7) $y = \exp\left(2\sin(x^2)+\frac{1}{x}\right)$

$$y' = \exp\left(2\sin(x^2)+\frac{1}{x}\right)\cdot\left(2\cos(x^2)\cdot 2x-\frac{1}{x^2}\right)$$

$$= \left(4x\cos(x^2)-\frac{1}{x^2}\right)\exp\left(2\sin(x^2)+\frac{1}{x}\right)$$

---

8) $y = \cos^2(x^3-5x)$ setze $z = x^3-5x$

$z' = 3x^2-5$

$$\Rightarrow y = \cos^2 z = (\cos z)^2$$

$$y' = 2(\cos z)^{2-1}\cdot(\cos z)'$$

$$= 2\cos z\cdot(\cos' z)\cdot z'$$

$$= 2\cos z\cdot(-\sin z)(3x^2-5)$$

$$= -2(3x^2-5)\sin z\cos z$$

$$= (10-6x^2)\sin(x^3-5x)\cos(x^3-5x)$$

Afg 4

1) $$y = \frac{x}{(1-x^3)^2}$$

$$y' = \frac{1\cdot(1-x^3)^2 - x\cdot 2(1-x^3)(-3x^2)}{(1-x^3)^4}$$

$$= \frac{\cancel{(1-x^3)}\left((1-x^3)+6x^3\right)}{(1-x^3)^{\cancel{4}\,3}} = \frac{5x^3+1}{(1-x^3)^3}$$

---

2) $$y = \frac{x+1}{\sqrt{2x-1}} = \frac{x+1}{(2x-1)^{\frac{1}{2}}}$$

$$y' = \frac{1\cdot(2x-1)^{\frac{1}{2}} - (x+1)\cdot\frac{1}{2}(2x-1)^{-\frac{1}{2}}\cdot 2}{2x-1}$$

$$\underset{\text{erweitern}}{\overset{\text{mit } (2x-1)^{\frac{1}{2}}}{=}} \frac{2x-1-(x+1)}{(2x-1)(2x-1)^{1/2}} = \frac{2x-1-x-1}{(2x-1)^{3/2}}$$

$$= \frac{x-2}{\sqrt{(2x-1)^3}}$$

---

3) $$y = \frac{x}{\sqrt{x^2+1}} = \frac{x}{(x^2+1)^{1/2}}$$

$$y' = \frac{1\cdot(x^2+1)^{1/2} - x\cdot\frac{1}{2}(x^2+1)^{-\frac{1}{2}}\cdot 2x}{x^2+1}$$

$$\underset{\text{erweitern}}{\overset{\text{mit } (x^2+1)^{1/2}}{=}} \frac{(x^2+1)-x^2}{(x^2+1)(x^2+1)^{1/2}} = \frac{1}{(x^2+1)^{3/2}} = \frac{1}{\sqrt{(x^2+1)^3}}$$

zu Afg 4

4) $y = \sin\left(\frac{2x-3}{1-4x}\right)$ setze $z = \frac{2x-3}{1-4x}$

dann ist $y = \sin z$ und

$$z' = \frac{2(1-4x)-(2x-3)(-4)}{(1-4x)^2}$$

$$= \frac{2(1-4x)+4(2x-3)}{(1-4x)^2}$$

$$= \frac{2-8x+8x-12}{(1-4x)^2} = -\frac{10}{(1-4x)^2}$$

Nun gilt:

$$y' = (\sin' z)\cdot z' = z'\cdot \cos z$$

$$= -\frac{10}{(1-4x)^2}\cdot\cos\left(\frac{2x-3}{1-4x}\right)$$

---

5) $y = \ln\left(\frac{x+2}{1-x}\right)$, setze $z = \frac{x+2}{1-x}$

dann ist $y = \ln z$ und $\frac{1}{z} = \frac{1-x}{x+2}$

$$z' = \frac{1\cdot(1-x)-(x+2)\cdot(-1)}{(1-x)^2} = \frac{(1-x)+(x+2)}{(1-x)^2} = \frac{3}{(1-x)^2}$$

Jetzt folgt:

$$y' = (\ln' z)\cdot z' = \frac{1}{z}\cdot z'$$

$$= \frac{1-x}{x+2}\cdot\frac{3}{(1-x)^2} = \frac{3}{(x+2)(1-x)}$$

zu Afg 4

6) $y = \sqrt{\frac{2x-1}{x^2+1}}$ setze $\boxed{z = \frac{2x-1}{x^2+1}}$

dann ist $\boxed{y = \sqrt{z}}$, $\boxed{\frac{1}{z} = \frac{x^2+1}{2x-1}}$

$$z' = \frac{2(x^2+1) - (2x-1)\cdot 2x}{(x^2+1)^2} = \frac{2x^2+2-4x^2+2x}{(x^2+1)^2}$$

$$\boxed{z' = \frac{2(x-x^2+1)}{(x^2+1)^2}}$$

Nun folgt:

$$y' = \frac{1}{2\sqrt{z}} \cdot z' = \frac{1}{2} \cdot z' \cdot \sqrt{\frac{1}{z}}$$

$$y' = \frac{x-x^2+1}{(x^2+1)^2} \cdot \sqrt{\frac{x^2+1}{2x-1}}$$

---

7) $y = \arccos x = \cos^{-1} x \iff \cos y = x \;\Big|\; \frac{d}{dx}$

$\iff (\cos' y) \cdot y' = 1$

$\iff y' = \frac{1}{\cos' y} = -\frac{1}{\sin y} = -\frac{1}{\sqrt{1-\cos^2 y}}$

$= -\frac{1}{\sqrt{1-x^2}}$

---

zu Afg 4

8) $y = x^{\tan x} \mid \ln \iff \ln y = \ln x^{\tan x}$

$\ln y = \tan x \cdot \ln x \mid \frac{d}{dx} \iff \frac{y'}{y} = (\tan x \cdot \ln x)'$

$\frac{y'}{y} = \tan' x \cdot \ln x + \tan x \cdot \ln' x$

$\frac{y'}{y} = (1+\tan^2 x) \ln x + \tan x \cdot \frac{1}{x} \quad \mid \cdot y$

$y' = \left((1+\tan^2 x) \ln x + \frac{\tan x}{x} \right) \cdot x^{\tan x}$

---

9) $y = (3x-1)^{\sqrt{x}} \mid \ln \iff \ln y = \ln (3x-1)^{\sqrt{x}}$

$\ln y = \sqrt{x} \cdot \ln(3x-1) \mid \frac{d}{dx}$

$\frac{y'}{y} = (\sqrt{x})' \cdot \ln(3x-1) + \sqrt{x} \cdot [\ln(3x-1)]'$

$= \frac{1}{2\sqrt{x}} \cdot \ln(3x-1) + \sqrt{x} \cdot \frac{3}{3x-1} \quad \mid \cdot y$

$y' = \left(\frac{\ln(3x-1)}{2\sqrt{x}} + \frac{3\sqrt{x}}{3x-1} \right) \cdot (3x-1)^{\sqrt{x}}$