

Lösungen zu Termumformungen

⑦

Aufg 1 (Faktorisieren)

$$\begin{aligned} 1) \quad 2x - 6xy - a + 3ay &= 2x(1-3y) - a(1-3y) \\ &= (2x-a)(1-3y) \end{aligned}$$

$$\begin{aligned} 2) \quad xy - 2x + y - 2 &= x(y-2) + (y-2) \\ &= (x+1)(y-2) \end{aligned}$$

$$\begin{aligned} 3) \quad 6axy + 2abx - 15a^2b - 3a^2y &= 3a(2xy + 10bx - 5ab - ay) \\ &= 3a(2x(y+5b) - a(y+5b)) = 3a(2x-a)(y+5b) \end{aligned}$$

$$\begin{aligned} 4) \quad 21ax - 15ay + 28bx - 20by \\ &= 3a(7x-5y) + 4b(7x-5y) \\ &= (3a+4b)(7x-5y) \end{aligned}$$

$$\begin{aligned} 5) \quad 112mp^2 - 63mq^2 &= 7m(16p^2 - 9q^2) \\ &= 7m(4p+3q)(4p-3q) \end{aligned}$$

3. bin. Formel

Tipp: Die Zahlen 16 und 9 mit dem Taschenrechner
(Bruchfunktion $\frac{\square}{\square}$) ermitteln $\frac{112}{63} = \frac{16}{9}$!

$$\begin{aligned} 6) \quad 5x^2 + 20xy + 20y^2 &= 5(x^2 + 4xy + 4y^2) \\ &= 5(x+2y)^2 \end{aligned}$$

nächste Seite! →

$$\begin{aligned}
 7) & 100a^2r^2 - 25a^2s^2 - 120abr^2 + 30abs^2 + 36b^2r^2 - 9b^2s^2 \quad (2) \\
 &= 25a^2(4r^2 - s^2) - 30ab(4r^2 - s^2) + 9b^2(4r^2 - s^2) \\
 &= (25a^2 - 30ab + 9b^2)(4r^2 - s^2) \\
 &= (5a - 3b)^2(4r + s)(4r - s)
 \end{aligned}$$

$$\begin{aligned}
 8) & 2amx - 2anx + amy - any - 6bmx + 6bnx - 3bmy + 3bny \\
 &= a(2mx - 2nx + my - ny) - 3b(2mx - 2nx + my - ny) \\
 &= (a - 3b)[2x(m - n) + y(m - n)] \\
 &= (a - 3b)(2x + y)(m - n)
 \end{aligned}$$

$$\begin{aligned}
 9) & 66amx - 78anx + 231bmx - 273bnx - 110amy + 130any - 385bmy \\
 & \quad + 455bny \\
 &= 6ax(11m - 13n) + 21bx(11m - 13n) - 10ay(11m - 13n) - 35by(11m - 13n) \\
 &= (6ax + 21bx - 10ay - 35by)(11m - 13n) \\
 &= (3x(2a + 7b) - 5y(2a + 7b))(11m - 13n) \\
 &= (3x - 5y)(2a + 7b)(11m - 13n)
 \end{aligned}$$

$$\begin{aligned}
 10) & (4a - 5b)(3m - 2p) + (3a + 5b)(3m - 2p) \\
 &= ((4a - 5b) + (3a + 5b))(3m - 2p) = 7a(3m - 2p)
 \end{aligned}$$

Bem: Vereinfachen durch Ausmultiplizieren wäre hier zu rechenintensiv!

$$\begin{aligned}
 11) & (9a^2 + 12a + 4) - 16b^2 \stackrel{\substack{1. \text{ Bin.} \\ \text{Formel}}}{=} (3a + 2)^2 - 16b^2 \\
 & \stackrel{\substack{3. \text{ Bin.} \\ \text{Formel}}}{=} (3a + 2 + 4b)(3a + 2 - 4b)
 \end{aligned}$$

$$\begin{aligned}
 1) \quad & \frac{2a^3 - 2a^2c + 2a^2b - 2abc}{(a^2x - a^3 - acx + a^2c) + (abx - a^2b - bcx + abc)} \\
 &= \frac{2a(a^2 - ac + ab - bc)}{a(ax - a^2 - cx + ac) + b(ax - a^2 - cx + ac)} \\
 &= \frac{2a(a(a-c) + b(a-c))}{(a+b)(a(x-a) - c(x-a))} = \frac{2a(a+b)(a-c)}{(a+b)(a-c)(x-a)} \\
 &= \frac{2a}{x-a}
 \end{aligned}$$

$$\begin{aligned}
 2) \quad & \frac{63x^2 + 42hx + 7h^2}{15hx^2 + 5h^2x - 30mhx - 10mh^2} \\
 &= \frac{7(9x^2 + 6hx + h^2)}{5h(3x^2 + hx - 6mx - 2mh)} = \frac{7(3x+h)^2}{5h(x(3x+h) - 2m(3x+h))} \\
 &= \frac{7(3x+h)^2}{5h(x-2m)(3x+h)} = \frac{7(3x+h)}{5h(x-2m)}
 \end{aligned}$$

$$\begin{aligned}
 3) \quad & \frac{12ap^2 + 36bp^2 - 12apq - 36bpq + 3a^2q + 36q^2}{2ap + 6bp - aq - 36q} \\
 &= \frac{3(4ap^2 + 12bp^2 - 4apq - 12bpq + a^2q + 36q^2)}{2p(a+3b) - q(a+3b)} \\
 &= \frac{3(4p^2(a+3b) - 4pq(a+3b) + q^2(a+3b))}{(2p-q)(a+3b)} \\
 &= \frac{3(4p^2 - 4pq + q^2)(a+3b)}{(2p-q)(a+3b)} = \frac{3(2p-q)^2(a+3b)}{(2p-q)(a+3b)} = 3(2p-q)
 \end{aligned}$$

$$\begin{aligned}
 4) \quad & \frac{3m^3 - 3m}{3am + 3a + 27mx + 27x} = \frac{3m(m^2 - 1)}{3(a + 7m + 7x)} \\
 & = \frac{3m(m^2 - 1)}{3(a(m+1) + 7x(m+1))} = \frac{3m(m+1)(m-1)}{3(a+7x)(m+1)} \\
 & = \frac{m(m-1)}{a+7x}
 \end{aligned}$$

$$\begin{aligned}
 5) \quad & \frac{4m^2x - 25n^2x - 16m^2y + 100n^2y}{12m^2x - 75n^2x} \\
 & = \frac{x(4m^2 - 25n^2) - 4y(4m^2 - 25n^2)}{3x(4m^2 - 25n^2)} \\
 & = \frac{(x-4y)(4m^2 - 25n^2)}{3x(4m^2 - 25n^2)} = \frac{x-4y}{3x}
 \end{aligned}$$

Aufg 3 (Brüche addieren u subtrahieren)

$$\begin{aligned}
 7) \quad & \frac{3ab - 1}{3ax - 6ay} + \frac{2b - 3}{4y - 2x} = \frac{3ab - 1}{3a(x - 2y)} + \frac{2b - 3}{2(2y - x)} \\
 & = \frac{2(3ab - 1) - 3a(2b - 3)}{6a(x - 2y)} = \frac{6ab - 2 - 6ab + 9a}{6a(x - 2y)} \\
 & = \frac{9a - 2}{6a(x - 2y)}
 \end{aligned}$$

(5)

$$2) \frac{3b}{x+1} + \frac{b+z}{a-y} - \frac{3by - b + 2xz}{xy - ax + y - a = x(y-a) + (y-a)}$$

$$= \frac{3b}{x+1} + \frac{b+z}{a-y} + \frac{3by - b + 2xz}{(x+1)(a-y)}$$

$$= \frac{3b(a-y) + (b+z)(x+1) + 3by - b + 2xz}{(x+1)(a-y)}$$

$$= \frac{3ab - 3by + bx + b + xz + z + 3by - b + 2xz}{(x+1)(a-y)}$$

$$= \frac{3ab + bx + 3xz + z}{(x+1)(a-y)}$$

$$3) \frac{a - 6by}{3y^2 - 3xy} - \frac{4ab - 1}{2ax - 2ay} = \frac{a - 6by}{3y(y-x)} - \frac{4ab - 1}{2a(x-y)}$$

$$= \frac{2a(a - 6by) + 3y(4ab - 1)}{6ay(y-x)} = \frac{2a^2 - 12aby + 12aby - 3y}{6ay(y-x)}$$

$$= \frac{2a^2 - 3y}{6ay(y-x)}$$

$$4) \frac{a}{9a^2 - 4b^2} + \frac{b}{9a^2 - 12ab + 4b^2} - \frac{1}{3a - 2b}$$

$$= \frac{a}{(3a+2b)(3a-2b)} + \frac{b}{(3a-2b)^2} - \frac{1}{3a-2b}$$

$$= \frac{a(3a-2b) + b(3a+2b) - (9a^2 - 4b^2)}{(3a+2b)(3a-2b)^2}$$

$$= \frac{3a^2 - 2ab + 3ab + 2b^2 - 9a^2 + 4b^2}{(3a+2b)(3a-2b)^2} = \frac{6b^2 + ab - 6a^2}{(3a+2b)(3a-2b)^2}$$

Aufg 4 (Brüche multiplizieren u. dividieren)

6

$$1) \left(\frac{1}{m} + \frac{1}{n}\right)(m+n) = \frac{n+m}{mn} \cdot (m+n) = \frac{(m+n)^2}{mn}$$

$$2) (x+y) : \left(\frac{1}{x} + \frac{1}{y}\right) = \frac{x+y}{1} : \frac{y+x}{xy} = \frac{x+y}{1} \cdot \frac{xy}{x+y} = xy$$

$$3) \frac{a+b}{ab} : \left(2 + \frac{a}{b} + \frac{b}{a}\right) = \frac{a+b}{ab} : \frac{2ab+a^2+b^2}{ab} = \frac{a+b}{ab} \cdot \frac{ab}{(a+b)^2} \\ = \frac{1}{a+b}$$

$$4) \left(\frac{m}{n} - \frac{r}{s}\right) : \left(\frac{m}{n} + \frac{r}{s}\right) = \frac{ms-rn}{ns} : \frac{ms+rn}{ns} \\ = \frac{ms-rn}{ns} \cdot \frac{ns}{ms+rn} = \frac{ms-rn}{ms+rn}$$

$$5) \left(\frac{mp}{nq} + \frac{np}{mq}\right) : \frac{p}{q} = \frac{p}{q} \left(\frac{m}{n} + \frac{n}{m}\right) : \frac{p}{q} = \frac{m}{n} + \frac{n}{m} \\ = \frac{m^2+n^2}{mn}$$

$$6) \left(\frac{a}{2} - \frac{b}{3}\right) : (2b-3a) = \frac{3a-2b}{6} \cdot \frac{1}{2b-3a} = -\frac{(2b-3a)}{6(2b-3a)} \\ = -\frac{1}{6}$$

Auf 5 (Polynomdivision)

(7)

$$1) (5x^3 - 2x^2 - x - 2) : (x - 1) = 5x^2 + 3x + 2$$

$$\underline{-(5x^3 - 5x^2)}$$

$$\begin{array}{r} 3x^2 - x - 2 \\ \underline{-(3x^2 - 3x)} \\ 2x - 2 \\ \underline{\quad\quad} \end{array}$$

mit Horner-Schema:

	5	-2	-1	-2
1	0	5	3	2
Σ	5	3	2	0

$$2) (2x^4 - 3x^3 - 10x^2 + 16x - 3) : (x^2 + x - 3) = 2x^2 - 5x + 1$$

$$\underline{-(2x^4 + 2x^3 - 6x^2)}$$

$$\begin{array}{r} -5x^3 - 4x^2 + 16x - 3 \\ \underline{-(-5x^3 - 5x^2 + 15x)} \end{array}$$

$$\underline{\quad\quad x^2 + x - 3}$$

$$3) (15x^5 - 11x^4 + 24x^3 + 27x^2 - 11x + 7) : (3x^2 - 4x + 7) = 5x^3 + 2x^2 - x + 1$$

$$\underline{-(15x^5 - 20x^4 + 35x^3)}$$

$$\begin{array}{r} 6x^4 - 11x^3 + 27x^2 - 11x + 7 \\ \underline{-(6x^4 - 8x^3 + 14x^2)} \end{array}$$

$$\begin{array}{r} -3x^3 + 7x^2 - 11x + 7 \\ \underline{-(-3x^3 + 4x^2 - 7x)} \end{array}$$

$$\underline{\quad\quad 3x^2 - 4x + 7}$$

4) $(x^3 + 7) : (x + 7) = x^2 - x + 7$

$$\begin{array}{r}
 -(x^3 + x^2) \\
 \hline
 -x^2 + 7 \\
 -(-x^2 - x) \\
 \hline
 x + 7 \\
 \hline
 \text{-----}
 \end{array}$$

mit Horner - Schema:

	x^3	x^2	x	x^0
	1	0	0	7
-7	0	-7	7	-7
Σ	1	-7	7	0
	x^2	x	x^0	

5) $(4x^2 + 3x + 2) : (x + 7) = 4x - 7 + \frac{3}{x + 7}$

$$\begin{array}{r}
 -(4x^2 + 4x) \\
 \hline
 -x + 2 \\
 -(-x - 7) \\
 \hline
 \text{Rest } 3
 \end{array}$$

mit Horner:

	4	3	2
-7	0	-4	1
Σ	4	-1	3
	x	x^0	Rest

6) $(3x^3 - 5x^2 + 23x - 35) : (x^2 + 7) = 3x - 5 + \frac{2x}{x^2 + 7}$

$$\begin{array}{r}
 -(3x^3 + 21x) \\
 \hline
 -5x^2 + 2x - 35 \\
 -(-5x^2 - 35) \\
 \hline
 \text{Rest } 2x
 \end{array}$$