Lösungen zu Blatt 7a

Afg 1

a) $f(x) = x^3 - x^2 + 1 \Rightarrow f'(x) = 3x^2 - 2x$

Tangente t an f bei $x = -1$ bestimmen:

$t(x) = f'(-1)(x+1) + f(-1)$

$= 5(x+1) + (-1) \Rightarrow \boxed{t(x) = 5x + 4}$

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b) $g(x) = 1 - x^2 \Rightarrow g'(x) = -2x$

Tangenten t durch $P(-1|9)$ an g bestimmen:

Sei $t(x) = ax + b \Rightarrow 9 = t(-1) = -a + b$

$\Leftrightarrow b = a + 9$

$\Rightarrow \boxed{t(x) = ax + a + 9}$

Löse das Gleichungssystem

$$\begin{array}{lll} \text{I.} & g(x) = t(x) & \\ \text{II} & g'(x) = a & \end{array} \Leftrightarrow \begin{array}{ll} \text{I} & 1 - x^2 = ax + a + 9 \\ \text{II} & -2x = a \end{array}$$

Setze II in I ein:

$1 - x^2 = -2x^2 - 2x + 9 \Leftrightarrow x^2 + 2x - 8 = 0$

$x_{1/2} = \frac{-2 \pm \sqrt{4 + 32}}{2} = \frac{-2 \pm 6}{2}$

$x_1 = 2 \overset{\text{II}}{\Rightarrow} a_1 = -4 \Rightarrow \boxed{t_1(x) = -4x + 5}$ $\boxed{B_1(2|-3)}$

$x_2 = -4 \overset{\text{II}}{\Rightarrow} a_2 = 8 \Rightarrow \boxed{t_2(x) = 8x + 17}$ $\boxed{B_2(-4|-15)}$

c) $h(x) = \frac{x^2-1}{x-3}$, $t(x) = ax+b$

Tangente t an h mit Steigung -1 bestimmen:

$$h'(x) = \frac{2x(x-3)-(x^2-1)}{(x-3)^2} = \frac{2x^2-6x-x^2+1}{(x-3)^2}$$

$$h'(x) = \frac{x^2-6x+1}{(x-3)^2} = -1 \quad | \cdot (x-3)^2$$

$$\Leftrightarrow x^2-6x+1 = -(x-3)^2$$

$$\Leftrightarrow x^2-6x+1 = -x^2+6x-9 \Leftrightarrow 2x^2-12x+10=0$$

$$\Leftrightarrow x^2-6x+5=0 \qquad x_{1/2} = \frac{6 \pm \sqrt{36-20}}{2} = \frac{6 \pm 4}{2}$$

$$x_1 = 5 \Rightarrow \boxed{B_1(5/12)} \Rightarrow \boxed{t_1(x) = -x+17}$$

$$x_2 = 1 \Rightarrow \boxed{B_2(1/0)} \Rightarrow \boxed{t_2(x) = -x+1}$$

Afg 2 Nullstellen auf 6 Nachkommastellen genau bestimmen:

a) $f(x) = x^2-2$, Startwert $x_0 = 1{,}5$

mit $f(x_n) = 0{,}\overset{1}{0}\overset{2}{0}\overset{3}{0}\overset{4}{0}\overset{5}{0}z$, $z \neq 0$

$f'(x) = 2x \Rightarrow \boxed{x_{n+1} = x_n - \frac{x_n^2-2}{2x_n}}$ Newton-verfahren

n	x_n	$f(x_n)$
0	1,5	0,25
1	1,416667	0,007
2	1,414216	0,000007

somit $\boxed{x_2 = 1{,}414216}$

b) $g(x) = 4x^3 - 6x^2 + 1$, Startwert $x_0 = -1$

mit $g(x_n) = 0{,}\overset{1}{0}\overset{2}{0}\overset{3}{0}\overset{4}{0}\overset{5}{0}z$, $z \neq 0$

$g'(x) = 12x^2 - 12x \implies \boxed{x_{n+1} = x_n - \dfrac{4x_n^3 - 6x_n^2 + 1}{12x_n(x_n - 1)}}$

n	x_n	$g(x_n)$
0	-1	-9
1	$-0{,}625$	$-2{,}3$
2	$-0{,}434615$	$-0{,}5$
3	$-0{,}372905$	$-0{,}04$
4	$-0{,}366106$	$-0{,}0005$
5	$-0{,}366025$	$0{,}000002$

somit $\boxed{x_5 = -0{,}366025}$

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c) $h(x) = e^x - 3x$, Startwert $x_0 = 1{,}3$

mit $h(x_n) = 0{,}\overset{1}{0}\overset{2}{0}\overset{3}{0}\overset{4}{0}\overset{5}{0}\overset{6}{0}z$, $z \neq 0$

$h'(x) = e^x - 3 \implies \boxed{x_{n+1} = x_n - \dfrac{e^{x_n} - 3x_n}{e^{x_n} - 3}}$

n	x_n	$h(x_n)$
0	$1{,}3$	$-0{,}2$
1	$1{,}644695$	$0{,}2$
2	$1{,}532122$	$0{,}03$
3	$1{,}512699$	$0{,}0009$
4	$1{,}512135$	$0{,}0000007$

somit $\boxed{x_4 = 1{,}512135}$

Afg 3

$$f(x) = 2 - \frac{1}{2}(x-3)^2$$

Krümmungsradius r und Mittelpunkte M(a|b) der Krümmungskreise an f bei $x_0 = 3$ und $x_0 = 2$ bestimmen:

$$f'(x) = -x+3 \quad ; \quad f''(x) = -1$$

$\underline{x_0 = 3}$ $\Rightarrow$ $y = f(3) = 2$, $y' = f'(3) = 0$; $y'' = -1$

$$a = x_0 - \frac{y'(1+(y')^2)}{y''} = 3 - \frac{0\cdot(1+0)}{-1} = 3$$

$$b = \frac{1+(y')^2}{y''} + y = \frac{1+0}{-1} + 2 = 1$$

$$\Rightarrow \boxed{M(3|1)}$$

$$r = \frac{(1+(y')^2)^{1,5}}{|y''|} = \frac{(1+0)^{1,5}}{1} = \boxed{1}$$

$\underline{x_0 = 2}$ $\Rightarrow$ $y = f(2) = 1,5$; $y' = f'(2) = 1$; $y'' = -1$

$$a = 2 - \frac{1+1}{-1} = 2+2 = 4$$

$$b = \frac{1+1}{-1} + 1,5 = -2+1,5 = -0,5$$

$$\Rightarrow \boxed{M(4|-0,5)}$$

$$r = \frac{(1+1)^{1,5}}{1} = 2^{\frac{3}{2}} = \boxed{\sqrt{8} \approx 2,83}$$