Lösungen (Gleichungen / Ungleichungen) 1

Afg 1

1) $1+\frac{x-11}{2x-10}=\frac{2x+1}{3x-15} \Leftrightarrow 1+\frac{x-11}{2(x-5)}=\frac{2x+1}{3(x-5)} \quad |\cdot 6(x-5)$

$\boxed{\mathbb{D}=\mathbb{R}\setminus\{5\}}$

$6(x-5)+3(x-11)=2(2x+1)$

$6x-30+3x-33=4x+2$

$9x-63=4x+2 \Leftrightarrow 5x=65 \Leftrightarrow x=13$

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2) $\frac{9}{x-5}-\frac{28}{35-7x}=\frac{5}{x-9}$

$\Leftrightarrow \frac{9}{x-5}+\frac{28}{7(x-5)}=\frac{5}{x-9}$; $\boxed{\mathbb{D}=\mathbb{R}\setminus\{5;9\}}$

$\Leftrightarrow \frac{9}{x-5}+\frac{4}{x-5}=\frac{5}{x-9}$

$\Leftrightarrow \frac{13}{x-5}=\frac{5}{x-9} \quad |\cdot(x-5)(x-9)$

$\Leftrightarrow 13(x-9)=5(x-5) \Leftrightarrow 13x-117=5x-25$

$\Leftrightarrow 8x=92 \Leftrightarrow x=\frac{92}{8}=\frac{23}{2}=11\frac{1}{2}$

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3) $\frac{4x-5}{3x+3}+\frac{3x+4}{5-5x}=\frac{11x^2-69x+58}{15x^2-15}$ $\boxed{\mathbb{D}=\mathbb{R}\setminus\{1;-1\}}$

$\Leftrightarrow \frac{4x-5}{3(x+1)}-\frac{3x+4}{5(x-1)}=\frac{11x^2-69x+58}{15(x+1)(x-1)} \quad |\cdot 15(x+1)(x-1)$

$5(4x-5)(x-1)-3(3x+4)(x+1)=11x^2-69x+58$

$5(4x^2-9x+5)-3(3x^2+7x+4)=11x^2-69x+58$

$20x^2-45x+25-9x^2-21x-12=11x^2-69x+58$

$\cancel{11x^2}-66x+13=\cancel{11x^2}-69x+58 \Leftrightarrow 3x=45 \Leftrightarrow x=15$

4) $\frac{7x-13}{2x-1} - \frac{13x-28}{3-2x} = 10 - \frac{28x+43}{4x^2-8x+3}$ $\boxed{D = \mathbb{R} \setminus \{\frac{1}{2}; \frac{3}{2}\}}$ (2)

$\Leftrightarrow \frac{7x-13}{2x-1} + \frac{13x-28}{2x-3} = 10 - \frac{28x+43}{(2x-1)(2x-3)} \quad | \cdot (2x-1)(2x-3)$

$\Leftrightarrow (7x-13)(2x-3) + (13x-28)(2x-1) = 10(4x^2-8x+3) - (28x+43)$

$14x^2 - 47x + 39 + 26x^2 - 69x + 28 = 40x^2 - 80x + 30 - 28x - 43$

$40x^2 - 116x + 67 = 40x^2 - 108x - 13 \quad | -40x^2$

$67 - 116x = -108x - 13 \Leftrightarrow 8x = 80 \Leftrightarrow x = 10$

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5) $\frac{x}{x-2} - \frac{x-2}{x-3} = \frac{2}{5x-x^2-6}$ $\boxed{D = \mathbb{R} \setminus \{2; 3\}}$

$\Leftrightarrow \frac{x}{x-2} - \frac{x-2}{x-3} = -\frac{2}{(x-2)(x-3)} \quad | \cdot (x^2-5x+6)$

$x(x-3) - (x-2)^2 = -2$

$x^2 - 3x - x^2 + 4x - 4 = -2$

$x - 4 = -2 \Leftrightarrow x = 2 \notin D$ ↯

$\Rightarrow \mathbb{L} = \emptyset$

$\longrightarrow$

zu Afg 1

$\boxed{D = \mathbb{R} \setminus \{\frac{3}{4}; \frac{5}{4}\}}$

6) $$\frac{5x-17}{4x-3} + \frac{7(x-4)}{4x-5} = \frac{12(10x+73)}{32x-16x^2-15} + 3$$

$$\frac{5x-17}{4x-3} + \frac{7(x-4)}{4x-5} = 3 - \frac{12(10x+73)}{(4x-3)(4x-5)} \quad | \cdot (4x-3)(4x-5)$$

$$(5x-17)(4x-5) + 7(x-4)(4x-3) = 3(16x^2-32x+15) - 12(10x+73)$$

$$20x^2-93x+85+7(4x^2-19x+12) = 48x^2-96x+45-120x-876$$

$$20x^2-93x+85+28x^2-133x+84 = 48x^2-216x-831$$

$$48x^2-226x+169 = 48x^2-216x-831 \quad | -48x^2$$

$$\Leftrightarrow \quad 169-226x = -216x-831$$

$$\Leftrightarrow \quad 10x = 1000 \quad \Leftrightarrow \quad x = 100$$

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7) $$\frac{4x-1}{8x+12} - \frac{32-17x-12x^2}{12x^2-27} = \frac{15x-17}{10x-15}$$ $\boxed{D = \mathbb{R} \setminus \{\pm \frac{3}{2}\}}$

$$\frac{4x-1}{4(2x+3)} - \frac{32-17x-12x^2}{3(2x+3)(2x-3)} = \frac{15x-17}{5(2x-3)} \quad | \cdot 3 \cdot 4 \cdot 5(2x+3)(2x-3)$$

$$15(2x-3)(4x-1) - 20(32-17x-12x^2) = 12(2x+3)(15x-17)$$

$$15(8x^2-14x+3) - 640+340x+240x^2 = 12(30x^2+11x-51)$$

$$120x^2-210x+45-640+340x+240x^2 = 360x^2+132x-612$$

$$360x^2+130x-595 = 360x^2+132x-612$$

$$2x = 17 \quad \Leftrightarrow \quad x = 8{,}5$$

Afg 2 (Formeln umstellen) (4)

1) $c_1 m_1 (t_1 - t_m) = c_2 m_2 (t_m - t_2)$ $\boxed{\text{nach } t_m}$

$c_1 m_1 t_1 - c_1 m_1 t_m = c_2 m_2 t_m - c_2 m_2 t_2$

$c_1 m_1 t_m + c_2 m_2 t_m = c_1 m_1 t_1 + c_2 m_2 t_2$

$(c_1 m_1 + c_2 m_2) t_m = c_1 m_1 t_1 + c_2 m_2 t_2 \quad | : (\ldots)$

$$\boxed{t_m = \frac{c_1 m_1 t_1 + c_2 m_2 t_2}{c_1 m_1 + c_2 m_2}}$$

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2) $p_1 + \rho g h_1 + \frac{\rho}{2} v_1^2 = p_2 + \rho g h_2 + \frac{\rho}{2} v_2^2 \quad | \cdot 2$ $\boxed{\text{nach } \rho}$

$2 p_1 + 2 \rho g h_1 + \rho v_1^2 = 2 p_2 + 2 \rho g h_2 + \rho v_2^2$

$2 \rho g h_1 + \rho v_1^2 - 2 \rho g h_2 - \rho v_2^2 = 2 p_2 - 2 p_1$

$[2g(h_1 - h_2) + v_1^2 - v_2^2] \rho = 2(p_2 - p_1)$

$$\boxed{\rho = \frac{2(p_2 - p_1)}{2g(h_1 - h_2) + v_1^2 - v_2^2}}$$

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3) $v = a\left(c_1 - \frac{m_2(c_1 - c_2)}{m_1 + m_2}\right) \Leftrightarrow \frac{m_2(c_1 - c_2)}{m_1 + m_2} + \frac{v}{a} = c_1 \quad | \cdot a(m_1 + m_2)$

$\boxed{\text{nach } c_1}$

$a m_2 (c_1 - c_2) + v(m_1 + m_2) = a(m_1 + m_2) c_1$

$a m_2 c_1 - a m_2 c_2 + v(m_1 + m_2) = a(m_1 + m_2) c_1 \quad | - a m_2 c_1$

$a(m_1 + m_2 - m_2) c_1 = v(m_1 + m_2) - a m_2 c_2$

$a m_1 c_1 = v(m_1 + m_2) - a m_2 c_2 \Leftrightarrow \boxed{c_1 = \frac{v(m_1 + m_2) - a m_2 c_2}{a m_1}}$

5

4) $R - 2Q = \dfrac{R-Q}{Q+1} \quad \Big| \cdot (Q+1)$ $\boxed{\text{nach } R}$

$$(R-2Q)(Q+1) = R-Q$$

$$RQ + \cancel{R} - 2Q^2 - 2Q = \cancel{R} - Q \quad \Big| -R + 2Q + 2Q^2$$

$$RQ = 2Q^2 + Q \iff \boxed{R = 2Q+1}$$

$Q \neq 0$ vorausgesetzt

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5) $\dfrac{2A}{A-M} = 1 + \dfrac{A}{A+N} \quad \Big| \cdot (A-M)(A+N)$ $\boxed{\text{nach } A}$

$$2A(A+N) = (A-M)(A+N) + A(A-M)$$

$$2A^2 + 2AN = A^2 + AN - AM - MN + A^2 - AM$$

$$\cancel{2A^2} + 2AN = \cancel{2A^2} + AN - 2AM - MN \quad \Big| -AN + 2AM$$

$$AN + 2AM = -MN$$

$$(N+2M)A = -MN \iff \boxed{A = -\frac{MN}{2M+N}}$$

Afg 3 (Ungleichungen)

1) $\dfrac{2-x}{3} \geq \dfrac{x+3}{2} \quad \Big| \cdot 6 \iff 2(2-x) \geq 3(x+3)$

$$\iff 4 - 2x \geq 3x + 9 \iff 5x \leq -5 \iff x \leq -1$$

$$\mathbb{L} =]-\infty ; -1]$$

$\longrightarrow$

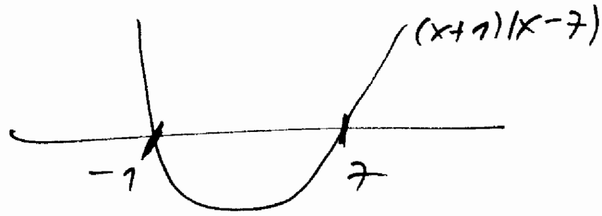
2) $\frac{3x+7}{x-3} < x \,\Big|\cdot(x-3)$ $\quad \mathbb{D} = \mathbb{R}\setminus\{3\}$ (6)

1. Fall: $x-3>0 \iff x>3$

$3x+7 < x(x-3) = x^2-3x$

$x^2-6x-7>0 \iff (x+1)(x-7)>0$

am besten graphisch lösen (Parabel nach oben geöffnet)



man sieht: $(x+1)(x-7)>0 \iff x<-1 \vee x>7$

$\Rightarrow \mathbb{L}_1 =]7;\infty[$ da auch $x>3$ gelten muss

2. Fall: $x-3<0 \iff x<3$

$3x+7 > x^2-3x \iff x^2-6x-7<0$

$\iff (x+1)(x-7)<0$

$\overset{\text{Graph}}{\iff} -1<x<7$

$\Rightarrow \mathbb{L}_2 =]-1;3[$

Insgesamt: $\mathbb{L} = \mathbb{L}_1 \cup \mathbb{L}_2 =]-1;3[\cup]7;\infty[$

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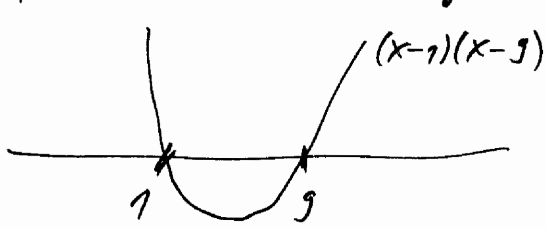
3) $9 \le x(10-x) \iff 9 \le 10x - x^2 \iff x^2-10x+9 \le 0$

$\iff (x-1)(x-9) \le 0$

Parabel nach oben geöffnet: $\iff 1 \le x \le 9$



$\Rightarrow \mathbb{L} = [1;9]$

4) $\frac{2x-1}{8x-4} > 0 \iff \frac{2x-1}{4(2x-1)} > 0$

$\iff \frac{1}{4} > 0$ (w) für $x \neq \frac{1}{2}$

$\Rightarrow \mathbb{L} = \mathbb{R} \setminus \{\frac{1}{2}\}$

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5) $x^3 > 4x \iff x^3 - 4x > 0$

$\iff x(x^2-4) > 0$

$\iff x(x+2)(x-2) > 0$

falls $x > 0 \Rightarrow x^2 - 4 > 0 \iff x > 2 \vee x < -2$ ~~$\}$~~

$\Rightarrow x > 2$

falls $x < 0 \Rightarrow x^2 - 4 < 0 \iff -2 < x < 2$

$\Rightarrow -2 < x < 0$

insgesamt: $-2 < x < 0 \vee x > 2$

$\mathbb{L} =]-2; 0[\cup]2; \infty[$

Kann man alternativ auch mit Vorzeichentabelle lösen:

	$x < -2$	$-2 < x < 0$	$0 < x < 2$	$x > 2$
x	$-$	$-$	$+$	$+$
$x+2$	$-$	$+$	$+$	$+$
$x-2$	$-$	$-$	$-$	$+$
$x(x+2)(x-2)$	$-$	$+$	$-$	$+$

6) $x+3 > \frac{4}{x+3} \quad | \cdot (x+3) \qquad ; \mathbb{D} = \mathbb{R} \setminus \{-3\}$

1. Fall: $x+3 > 0 \Leftrightarrow x > -3$

$(x+3)^2 > 4 \Leftrightarrow x+3 > 2 \quad \vee \quad x+3 < -2$

$\Leftrightarrow \boxed{x > -1} \quad \vee \quad x < -5$ ↯ zu $x > -3$

$\Rightarrow \mathbb{L}_1 =]-1; \infty[$

2. Fall: $x+3 < 0 \Leftrightarrow x < -3$

$(x+3)^2 < 4 \Leftrightarrow -2 < x+3 < 2$

$\Leftrightarrow -5 < x < -1$

da $x < -3 \Rightarrow \boxed{-5 < x < -3}$

$\Rightarrow \mathbb{L}_2 =]-5; -3[$

insgesamt $\mathbb{L} = \mathbb{L}_1 \cup \mathbb{L}_2 =]-5; -3[\cup]-1; \infty[$

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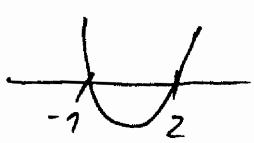
7) $x^2 \leq |x| + 2$

1. Fall: $x \geq 0$

$x^2 \leq x+2 \Leftrightarrow x^2 - x - 2 \leq 0 \Leftrightarrow (x+1)(x-2) \leq 0$

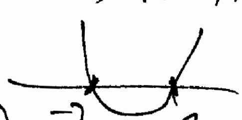


$\Leftrightarrow -1 \leq x \leq 2$

$\Rightarrow \mathbb{L}_1 = [0; 2]$

2. Fall: $x < 0$

$x^2 \leq -x+2 \Leftrightarrow x^2 + x - 2 \leq 0 \Leftrightarrow (x-1)(x+2) \leq 0$



$\Leftrightarrow -2 \leq x \leq 1$

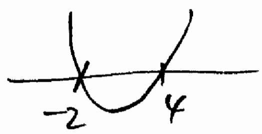
$\Rightarrow \mathbb{L}_2 = [-2; 0[$

$\mathbb{L} = \mathbb{L}_1 \cup \mathbb{L}_2 = [-2; 2]$

8) $|x^2-16| < 2x-8$

1. Fall: $x^2-16 \geq 0 \iff x \geq 4 \lor x \leq -4$

$x^2-16 < 2x-8 \iff x^2-2x-8 < 0$



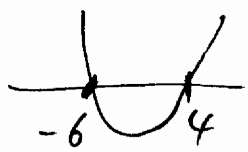
$\iff (x+2)(x-4) < 0$

$\iff -2 < x < 4 \quad \lightning$

$\implies \mathbb{L}_1 = \emptyset$

2. Fall: $x^2-16 < 0 \iff -4 < x < 4$

$16 - x^2 < 2x-8 \iff x^2+2x-24 > 0$



$\iff (x-4)(x+6) > 0$

$\iff x < -6 \lor x > 4 \quad \lightning$

$\implies \mathbb{L}_2 = \emptyset$

insgesamt: $\mathbb{L} = \mathbb{L}_1 \cup \mathbb{L}_2 = \emptyset$

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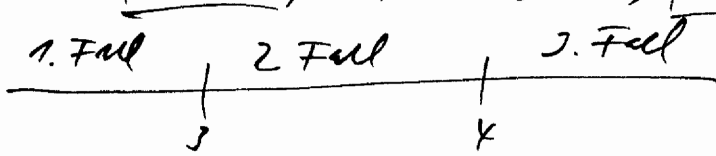
9) $|2x-6| + 7 \leq 9 - |x-4| \iff \boxed{|2x-6| + |x-4| \leq 2}$

$2x-6 \geq 0 \iff \boxed{x \geq 3}$; $\quad x-4 \geq 0 \iff \boxed{x \geq 4}$



1. Fall: $x < 3 \implies x < 4$

$6-2x+4-x \leq 2 \iff 10-3x \leq 2$

$\iff 3x \geq 8$

$\iff x \geq \frac{8}{3}$

$\implies \mathbb{L}_1 = [\frac{8}{3}; 3[$

$\longrightarrow$

2. Fall: $3 \leq x < 4$

$2x-6+4-x \leq 2 \iff x-2 \leq 2$

$\iff x \leq 4$

$\Rightarrow \mathbb{L}_2 = [3;4[$

3. Fall: $x \geq 4 \Rightarrow x > 3$

$2x-6+x-4 \leq 2 \iff 3x-10 \leq 2$

$\iff 3x \leq 12$

$\iff x \leq 4$

$\Rightarrow \mathbb{L}_3 = \{4\}$

Gesamte Lösungsmenge: $\mathbb{L} = \mathbb{L}_1 \cup \mathbb{L}_2 \cup \mathbb{L}_3 = [\frac{8}{3};4]$

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10) Zu lösen ist die Ungleichung

$$\boxed{|x-182| \leq 0{,}3x}$$

1. Fall: $x \geq 182$

$x-182 \leq 0{,}3x \iff 0{,}7x \leq 182 \iff x \leq 260$

$\mathbb{L}_1 = [182;260]$

2. Fall: $x < 182$

$182-x \leq 0{,}3x \iff 1{,}3x \geq 182 \iff x \geq 140$

$\mathbb{L}_2 = [140;182[$

gesamt: $\mathbb{L} = \mathbb{L}_1 \cup \mathbb{L}_2 = [140;260]$.

Dies gilt also für alle x mit $140 \leq x \leq 260$. ▨