Lösungen (Gleichungen)

Alg 1

1) $1 + \frac{x-11}{2x-10} = \frac{2x+1}{3x-15} \Leftrightarrow 1 + \frac{x-11}{2(x-5)} = \frac{2x+1}{3(x-5)} \quad | \cdot 6(x-5)$

$\mathbb{D} = \mathbb{R} \setminus \{5\}$

$6(x-5) + 3(x-11) = 2(2x+1)$

$6x - 30 + 3x - 33 = 4x + 2$

$9x - 63 = 4x + 2 \Leftrightarrow 5x = 65 \Leftrightarrow x = 13$

$\Rightarrow \mathbb{L} = \{13\}$

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2) $\frac{9}{x-5} - \frac{28}{35-7x} = \frac{5}{x-9}$

$\Leftrightarrow \frac{9}{x-5} + \frac{28}{7(x-5)} = \frac{5}{x-9} \quad ; \quad \boxed{\mathbb{D} = \mathbb{R} \setminus \{5; 9\}}$

$\Leftrightarrow \frac{9}{x-5} + \frac{4}{x-5} = \frac{5}{x-9}$

$\Leftrightarrow \frac{13}{x-5} = \frac{5}{x-9} \quad | \cdot (x-5)(x-9)$

$\Leftrightarrow 13(x-9) = 5(x-5) \Leftrightarrow 13x - 117 = 5x - 25$

$\Leftrightarrow 8x = 92 \Leftrightarrow x = \frac{92}{8} = \frac{23}{2} = 11\frac{1}{2}$

$\Rightarrow \mathbb{L} = \{11\frac{1}{2}\}$

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3) $\frac{4x-5}{3x+3} + \frac{3x+4}{5-5x} = \frac{11x^2 - 69x + 58}{15x^2 - 15}$

$\mathbb{D} = \mathbb{R} \setminus \{1; -1\}$

$\Leftrightarrow \frac{4x-5}{3(x+1)} - \frac{3x+4}{5(x-1)} = \frac{11x^2 - 69x + 58}{15(x+1)(x-1)} \quad | \cdot 15(x+1)(x-1)$

$5(4x-5)(x-1) - 3(3x+4)(x+1) = 11x^2 - 69x + 58$

$5(4x^2 - 9x + 5) - 3(3x^2 + 7x + 4) = 11x^2 - 69x + 58 \qquad \mathbb{L} = \{15\}$

$20x^2 - 45x + 25 - 9x^2 - 21x - 12 = 11x^2 - 69x + 58 \qquad \Uparrow$

$11x^2 - 66x + 13 = 11x^2 - 69x + 58 \Leftrightarrow 3x = 45 \Leftrightarrow x = 15$

4) $\frac{7x-13}{2x-1} - \frac{13x-28}{3-2x} = 10 - \frac{28x+43}{4x^2-8x+3}$ (2)

$\mathbb{D} = \mathbb{R} \setminus \{\frac{1}{2}; \frac{3}{2}\}$

$\Leftrightarrow \frac{7x-13}{2x-1} + \frac{13x-28}{2x-3} = 10 - \frac{28x+43}{(2x-1)(2x-3)} \quad | \cdot (2x-1)(2x-3)$

$\Leftrightarrow (7x-13)(2x-3) + (13x-28)(2x-1) = 10(4x^2-8x+3) - (28x+43)$

$14x^2 - 47x + 39 + 26x^2 - 69x + 28 = 40x^2 - 80x + 30 - 28x - 43$

$40x^2 - 116x + 67 = 40x^2 - 108x - 13 \quad | -40x^2$

$67 - 116x = -108x - 13 \Leftrightarrow 8x = 80 \Leftrightarrow x = 10$

$\Rightarrow \mathbb{L} = \{10\}$

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5) $\frac{x}{x-2} - \frac{x-2}{x-3} = \frac{2}{5x-x^2-6}$

$\mathbb{D} = \mathbb{R} \setminus \{2; 3\}$

$\Leftrightarrow \frac{x}{x-2} - \frac{x-2}{x-3} = -\frac{2}{(x-2)(x-3)} \quad | \cdot (x^2-5x+6)$

$x(x-3) - (x-2)^2 = -2$

$x^2 - 3x - x^2 + 4x - 4 = -2$

$x - 4 = -2 \Leftrightarrow x = 2 \notin \mathbb{D}$ ↯

$\Rightarrow \mathbb{L} = \emptyset$

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zu Alg 1

$\mathbb{D} = \mathbb{R} \setminus \{\frac{3}{4}; \frac{5}{4}\}$ (3)

6) $\frac{5x-17}{4x-3} + \frac{7(x-4)}{4x-5} = \frac{12(10x+77)}{32x-16x^2-15} + 3$

$\frac{5x-17}{4x-3} + \frac{7(x-4)}{4x-5} = 3 - \frac{12(10x+77)}{(4x-3)(4x-5)} \quad | \cdot (4x-3)(4x-5)$

$(5x-17)(4x-5) + 7(x-4)(4x-3) = 3(16x^2-32x+15) - 12(10x+77)$

$20x^2 - 93x + 85 + 7(4x^2 - 19x + 12) = 48x^2 - 96x + 45 - 120x - 876$

$20x^2 - 93x + 85 + 28x^2 - 133x + 84 = 48x^2 - 216x - 831$

$48x^2 - 226x + 169 = 48x^2 - 216x - 831 \quad | - 48x^2$

$\Leftrightarrow 169 - 226x = -216x - 831$

$\Leftrightarrow 10x = 1000 \Leftrightarrow x = 100 \Rightarrow \mathbb{L} = \{100\}$

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7) $\frac{4x-1}{8x+12} - \frac{32-17x-12x^2}{12x^2-27} = \frac{15x-17}{10x-15}$ $\quad \mathbb{D} = \mathbb{R} \setminus \{\pm \frac{3}{2}\}$

$\frac{4x-1}{4(2x+3)} - \frac{32-17x-12x^2}{3(2x+3)(2x-3)} = \frac{15x-17}{5(2x-3)} \quad | \cdot 3 \cdot 4 \cdot 5 (2x+3)(2x-3)$

$15(2x-3)(4x-1) - 20(32-17x-12x^2) = 12(2x+3)(15x-17)$

$15(8x^2 - 14x + 3) - 640 + 340x + 240x^2 = 12(30x^2 + 11x - 51)$

$120x^2 - 210x + 45 - 640 + 340x + 240x^2 = 360x^2 + 132x - 612$

$360x^2 + 130x - 595 = 360x^2 + 132x - 612$

$2x = 17 \Leftrightarrow x = 8{,}5 \Rightarrow \mathbb{L} = \{8{,}5\}$

Afg 2

1) $(2a-b)(x-a) = (a+x)(b-a)$

$2ax - 2a^2 - bx + \cancel{ab} = \cancel{ab} - a^2 + bx - ax$

$3ax - 2bx = a^2 \iff (3a-2b)x = a^2$

$\iff \boxed{x = \frac{a^2}{3a-2b}}$

2) $(m+2x)(x-n) + x = 2(x+c)(x-a) + m$

$mx - mn + \cancel{2x^2} - 2nx + x = \cancel{2x^2} - 2ax + 2cx - 2ac + m$

$2ax - 2cx + mx - 2nx + x = m - 2ac + mn$

$(2a - 2c + m - 2n + 1)x = m - 2ac + mn$

$\boxed{x = \frac{m(1+n) - 2ac}{2(a-c-n) + m + 1}}$

3) $(a-2x)^2 + bx = x - (b+2x)(b-2x)$

$a^2 - 4ax + \cancel{4x^2} + bx = x - b^2 + \cancel{4x^2}$

$a^2 + b^2 = 4ax - bx + x$

$a^2 + b^2 = (4a - b + 1)x \iff \boxed{x = \frac{a^2+b^2}{4a-b+1}}$

4) $(2m+3x)^2 - 4x = (4-3x)(2u-3x) + mx$

$4m^2 + 12mx + 9x^2 - 4x = 2u^2 - 3ux - 6ux + 9x^2 + mx$

$11mx + 8ux = 2u^2 - 4m^2$

$(11m+8u)x = 2u^2 - 4m^2 \iff \boxed{x = \frac{2u^2 - 4m^2}{11m+8u}}$

5) $c^2 + (a-x)^2 = (b-x)^2$

$c^2 + a^2 - 2ax + x^2 = b^2 - 2bx + x^2$

$2bx - 2ax = b^2 - a^2 - c^2$

$2x(b-a) = b^2 - a^2 - c^2 \iff \boxed{x = \frac{b^2 - a^2 - c^2}{2(b-a)}}$

6) $a - \frac{m+u}{x} = b - \frac{m-u}{x} \quad | + \frac{m-u}{x} - a$

$\frac{m-u}{x} - \frac{m+u}{x} = b - a$

$\frac{m-u-m-u}{x} = b-a \iff -\frac{2u}{x} = b-a \quad | \cdot x$

$-2u = (b-a)x \quad | : (b-a) \iff x = -\frac{2u}{b-a}$

$\boxed{x = \frac{2u}{a-b}}$

7) $\frac{a+b}{ax} = \frac{2b}{a^2} - \frac{a-b}{ax} \quad | + \frac{a-b}{ax}$

$\frac{a+b}{ax} + \frac{a-b}{ax} = \frac{2b}{a^2} \iff \frac{a+b+a-b}{ax} = \frac{2b}{a^2}$

$\frac{2a}{ax} = \frac{2b}{a^2} \quad | :2 \iff \frac{1}{x} = \frac{b}{a^2} \quad |$ Kehrwert

$$\boxed{x = \frac{a^2}{b}}$$

8) $\frac{a}{b+x} - m = \frac{c}{b+x} - n \quad | - \frac{c}{b+x} + m$

$\frac{a}{b+x} - \frac{c}{b+x} = m - n \iff \frac{a-c}{b+x} = m-n$

$a - c = (b+x)(m-n) \quad | : (m-n)$

$\frac{a-c}{m-n} = b+x \quad | -b \iff \boxed{x = \frac{a-c}{m-n} - b}$

9) $\frac{b-x}{a+x} + \frac{c-x}{a-x} = \frac{a(c-2x)}{a^2-x^2} = \frac{a(c-2x)}{(a+x)(a-x)} \quad | \cdot (a^2-x^2)$

$(a-x)(b-x) + (a+x)(c-x) = a(c-2x)$

$ab - ax - bx + x^2 + ac - ax + cx - x^2 = ac - 2ax$

$ab - \cancel{2ax} - bx + cx + \cancel{ac} - \cancel{ac} - \cancel{2ax} \quad | + bx - cx$

$ab = bx - cx \iff ab = (b-c)x$

$\iff \boxed{x = \frac{ab}{b-c}}$

10) $\frac{3x(x-a)}{3x-3a} = 3x-3a \iff \frac{3x(x-a)}{3(x-a)} = 3(x-a)$

$\iff 3x = 3(x-a) \mid :3 \iff x = x-a$

$\iff 0 = -a$

Da $x \neq a$, so ~~folgt~~

$$\mathbb{L} = \begin{cases} \mathbb{R}\setminus\{a\} & \text{für } a = 0 \\ \emptyset & \text{für } a \neq 0 \end{cases}$$

11) $2a+x = \frac{2a^2}{2a-x} + \frac{x(2a+x)}{x-2a} \quad \mid \cdot(2a-x)$

(*) $(2a+x)(2a-x) = 2a^2 - x(2a+x) \quad \mid + x(2a+x)$

$(2a+x)(2a-x) + x(2a+x) = 2a^2$

$(2a+x)(2a-x+x) = 2a^2$

$2a(2a+x) = 2a^2 \mid :2a \iff 2a+x = a$

$\iff \boxed{x = -a}$

Alternative:

(*) $4a^2 - x^2 = 2a^2 - 2ax - x^2$

$4a^2 = 2a^2 - 2ax \iff 2ax = -2a^2$

$\iff \boxed{x = -a}$

Lösungen zu Afg 3

1) $v = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$ nach m_1

$$v(m_1 + m_2) = m_1 v_1 + m_2 v_2$$

$$v m_1 + v m_2 = m_1 v_1 + m_2 v_2$$

$$v m_1 - m_1 v_1 = m_2 v_2 - v m_2$$

$$m_1 (v - v_1) = m_2 (v_2 - v)$$

$$\boxed{m_1 = \frac{m_2 (v_2 - v)}{v - v_1}}$$

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2) $v = \frac{m_1 v_1 + m_2 v_2 - (v_1 - v_2) m_2 g}{m_1 + m_2}$ nach m_2

$$v(m_1 + m_2) = m_1 v_1 + m_2 v_2 - (v_1 - v_2) m_2 g$$

$$v m_1 + v m_2 = m_1 v_1 + m_2 v_2 - v_1 m_2 g + v_2 m_2 g$$

$$v m_1 - m_1 v_1 = m_2 v_2 - v_1 m_2 g + v_2 m_2 g - v m_2$$

$$m_1 (v - v_1) = m_2 (v_2 - v_1 g + v_2 g - v)$$

$$\boxed{m_2 = \frac{m_1 (v - v_1)}{v_2 - v_1 g + v_2 g - v}}$$

3) $W = \gamma c \left(\frac{1}{v_1} - \frac{1}{v_2}\right)$ nach v_2

$$\frac{W}{\gamma c} = \frac{1}{v_1} - \frac{1}{v_2} \iff \frac{1}{v_2} = \frac{1}{v_1} - \frac{W}{\gamma c}$$

$$\frac{1}{v_2} = \frac{\gamma c - v_1 W}{v_1 \gamma c} \iff \boxed{v_2 = \frac{v_1 \gamma c}{\gamma c - v_1 W}}$$

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4) $\frac{p_1}{p_2} = \frac{T_0 + t_1}{T_0 + t_2}$ nach T_0

$$p_1 (T_0 + t_2) = p_2 (T_0 + t_1)$$

$$p_1 T_0 + p_1 t_2 = p_2 T_0 + p_2 t_1$$

$$p_1 T_0 - p_2 T_0 = p_2 t_1 - p_1 t_2$$

$$(p_1 - p_2) T_0 = p_2 t_1 - p_1 t_2$$

$$\boxed{T_0 = \frac{p_2 t_1 - p_1 t_2}{p_1 - p_2}}$$

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5) $H = \frac{2F}{\pi d \left(d - \sqrt{d^2 - D^2}\right)}$ nach D

$$d - \sqrt{d^2 - D^2} = \frac{2F}{\pi d H} \longrightarrow$$

$$\sqrt{d^2 - D^2} = d - \frac{2F}{\pi d H} \iff d^2 - D^2 = \left(d - \frac{2F}{\pi d H}\right)^2$$

$$D^2 = d^2 - \left(d - \frac{2F}{\pi d H}\right)^2$$

$$\boxed{D = \sqrt{d^2 - \left(d - \frac{2F}{\pi d H}\right)^2}}$$

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6) $$v = \sqrt{\frac{2p}{r\left[\left(\frac{A_1}{A_2}\right)^2 - 1\right]}}$$ nach A_1

$$v^2 = \frac{2p}{r\left[\left(\frac{A_1}{A_2}\right)^2 - 1\right]}$$

$$\left(\frac{A_1}{A_2}\right)^2 = \frac{2p}{r v^2}$$

$$\frac{A_1}{A_2} = \sqrt{\frac{2p}{r v^2}}$$

$$\boxed{A_1 = A_2 \sqrt{\frac{2p}{r v^2}}}$$

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7) $$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$ nach f_2

$$\frac{1}{f} = \frac{f_2 + f_1 - d}{f_1 f_2}$$

$$f_1 f_2 = f(f_2 + f_1 - d)$$

$$f_1 f_2 = f f_2 + f f_1 - f d$$

$$f_1 f_2 - f f_2 = f(f_1 - d)$$

$$f_2(f_1 - f) = f(f_1 - d)$$

$$\boxed{f_2 = \frac{f(f_1 - d)}{f_1 - f}}$$

8) $U_K = U_e - \frac{U_e R_i'}{R_i' + R_a}$ nach R_i'

$U_K (R_i' + R_a) = U_e (R_i' + R_a) - U_e R_i'$

$U_K R_i' + U_K R_a = U_e R_i' + U_e R_a - U_e R_i'$

$U_K R_i' = U_e R_a - U_K R_a$

$$\boxed{R_i' = \frac{R_a (U_e - U_K)}{U_K}}$$

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9) $R - 2Q = \frac{R - Q}{Q + 1}$ nach R

$(R - 2Q)(Q+1) = R - Q$

$RQ + R - 2Q^2 - 2Q = R - Q$

$RQ = 2Q^2 + Q \quad | : Q \;,\; Q \neq 0$

$$\boxed{R = 2Q + 1}$$

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10) $\frac{2A}{A - M} = 1 + \frac{A}{A + N}$ nach A

$\cdot (A - M)(A + N)$

$2A(A+N) = (A-M)(A+N) + A(A-M)$

$2A^2 + 2AN = A^2 + AN - AM - MN + A^2 - AM$

$2A^2 + 2AN = 2A^2 + AN - 2AM - MN$ →

$$AN + 2AM = -MN$$

$$A(N+2M) = -MN$$

$$\boxed{A = -\frac{MN}{N+2M}}$$

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11) $x = \dfrac{T_1^2 - T_2^2}{T_1^2 + T_2^2}$ nach T_1

$$x(T_1^2 + T_2^2) = T_1^2 - T_2^2$$

$$xT_1^2 + xT_2^2 = T_1^2 - T_2^2$$

$$xT_2^2 + T_2^2 = T_1^2 - xT_1^2$$

$$T_2^2(x+1) = T_1^2(1-x)$$

$$T_1^2 = \frac{T_2^2(x+1)}{1-x} \quad | \sqrt{}$$

$$\boxed{T_1 = T_2\sqrt{\frac{x+1}{1-x}}} \qquad T_2 > 0$$

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12) $a = \dfrac{d\sin(\alpha-\beta)}{\cos\beta}$ nach α

$$a\cos\beta = d\sin(\alpha-\beta)$$

$$\sin(\alpha-\beta) = \frac{a\cos\beta}{d} \Longrightarrow \boxed{\alpha = \sin^{-1}\left(\frac{a\cos\beta}{d}\right) + \beta}$$

13) $hf_0 = hf + \frac{m_e v^2}{2}$ nach h

$$hf_0 - hf = \frac{m_e v^2}{2}$$

$$h(f_0 - f) = \frac{m_e v^2}{2}$$

$$\boxed{h = \frac{m_e v^2}{2(f_0 - f)}}$$

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14) $R^2 = (R-d)^2 + r^2$ nach R

$$R^2 = R^2 - 2Rd + d^2 + r^2$$

$$0 = -2Rd + d^2 + r^2$$

$$2Rd = d^2 + r^2$$

$$\boxed{R = \frac{d^2 + r^2}{2d}}$$

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15) $m = \frac{m_0}{\sqrt{1-\beta^2}}$ nach β

$$\sqrt{1-\beta^2} = \frac{m_0}{m} \Rightarrow 1 - \beta^2 = \left(\frac{m_0}{m}\right)^2$$

$$\beta^2 = 1 - \left(\frac{m_0}{m}\right)^2$$

$$\boxed{\beta = \sqrt{1 - \left(\frac{m_0}{m}\right)^2}}$$

16) $\omega(L+r) = \frac{p-1}{\omega C}$ nach ω

$\omega^2 = \frac{p-1}{C(L+r)}$ | $\sqrt{\ }$

$$\boxed{\omega = \sqrt{\frac{p-1}{C(L+r)}}}$$

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17) $\frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1}$ nach x_1

$(y-y_1)(x_2-x_1) = (y_2-y_1)(x-x_1)$

$(y-y_1)x_2 - (y-y_1)x_1 = (y_2-y_1)x - (y_2-y_1)x_1$

$(y_2-y_1)x_1 - (y-y_1)x_1 = (y_2-y_1)x - (y-y_1)x_2$

$(y_2-y_1-y+y_1)x_1 = (y_2-y_1)x - (y-y_1)x_2$

$(y_2-y_.)x_1 = (y_2-y_1)x - (y-y_1)x_2$

$$\boxed{x_1 = \frac{(y_2-y_1)x - (y-y_1)x_2}{y_2-y}}$$

78) $I = \dfrac{G_1(G_2+G_3)}{G_1+G_2+G_3}$ nach G_1

$$I G_1 + I(G_2+G_3) = G_1(G_2+G_3)$$

$$I(G_2+G_3) = G_1(G_2+G_3) - I G_1$$

$$I(G_2+G_3) = G_1(G_2+G_3-I)$$

$$\boxed{G_1 = \frac{I(G_2+G_3)}{G_2+G_3-I}}$$

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79) $\dfrac{U^2}{R_i+R_V} = I^2(R_i+R_V)$ nach R_i

$$\frac{U^2}{I^2} = (R_i+R_V)^2 \Rightarrow \boxed{R_V = \frac{U}{I} - R_V}$$

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20) $P = R \cdot \dfrac{a(1-b)}{b(a-1)}$ nach a

$$Pb(a-1) = Ra(1-b)$$

$$Pba - Pb = Ra(1-b)$$

$$Pba - Ra(1-b) = Pb$$

$$a(Pb - R(1-b)) = Pb$$

$$\boxed{a = \frac{Pb}{Pb - R(1-b)}}$$