Lösungen LGS

82a)

$$\left(\begin{array}{ccc|c} 1 & 3 & -2 & 0 \\ 2 & -1 & 3 & 0 \\ -4 & 5 & -3 & 2 \end{array}\right) \xrightarrow[4\mathrm{I}+\mathrm{III}]{2\mathrm{I}-\mathrm{II}} \left(\begin{array}{ccc|c} 1 & 3 & -2 & 0 \\ 0 & 7 & -7 & 0 \\ 0 & 17 & -11 & 2 \end{array}\right)$$

$$\xrightarrow{\frac{1}{7}\mathrm{II}} \left(\begin{array}{ccc|c} 1 & 3 & -2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 17 & -11 & 2 \end{array}\right) \xrightarrow{17\mathrm{II}-\mathrm{III}} \left(\begin{array}{ccc|c} 1 & 3 & -2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & -6 & -2 \end{array}\right)$$

$\mathrm{III}: \ -6x_3 = -2 \iff \boxed{x_3 = \frac{1}{3}}$; $\mathrm{II}: \ x_2 - x_3 = 0 \iff \boxed{x_2 = \frac{1}{3}}$

$\mathrm{I}: \ x_1 + 3x_2 - 2x_3 = 0 \iff \boxed{x_1 = -\frac{1}{3}}$

---

b)

$$\left(\begin{array}{ccc|c} 1 & -1 & 2 & 11 \\ 3 & 1 & -1 & 1 \\ 4 & 0 & 1 & 12 \end{array}\right) \xrightarrow[4\mathrm{I}-\mathrm{III}]{3\mathrm{I}-\mathrm{II}} \left(\begin{array}{ccc|c} 1 & -1 & 2 & 11 \\ 0 & -4 & 7 & 32 \\ 0 & -4 & 7 & 32 \end{array}\right)$$

$$\xrightarrow{\mathrm{II}-\mathrm{III}} \left(\begin{array}{ccc|c} 1 & -1 & 2 & 11 \\ 0 & -4 & 7 & 32 \\ 0 & 0 & 0 & 0 \end{array}\right) \Longrightarrow \mathrm{II}: \ -4x_2 + 7x_3 = 32$$

setze $\boxed{x_3 = 4\lambda}$ mit $\lambda \in \mathbb{R}$ $\overset{\mathrm{II}}{\Longrightarrow}$ $\boxed{x_2 = 7\lambda - 8}$

$\mathrm{I}: \ x_1 - x_2 + 2x_3 = 11 \iff \boxed{x_1 = 3 - \lambda}$

Lösungsmenge = Schnittgerade dreier Ebenen:

$$\vec{x} = \begin{pmatrix} 3-\lambda \\ -8+7\lambda \\ 4\lambda \end{pmatrix} = \boxed{\begin{pmatrix} 3 \\ -8 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 7 \\ 4 \end{pmatrix}} \ ; \ \lambda \in \mathbb{R}$$

83a)

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & -4 \\ 1 & -1 & 1 & 0 \\ 2 & 1 & 3 & 7 \end{array}\right) \xrightarrow[2\mathrm{I}-\mathrm{III}]{\mathrm{I}-\mathrm{II}} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ 0 & 2 & 0 & 4 \\ 0 & 1 & -1 & 1 \end{array}\right)$$

$$\xrightarrow{\mathrm{II}-2\mathrm{III}} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ 0 & 2 & 0 & 4 \\ 0 & 0 & 2 & 2 \end{array}\right)$$; $\mathrm{III}: \ 2x_3 = 2 \iff \boxed{x_3 = 1}$

$\mathrm{II}: \ 2x_2 = 4 \iff \boxed{x_2 = 2}$; $\mathrm{I}: \ x_1 + x_2 + x_3 = 4 \iff \boxed{x_1 = 1}$

b) $\left(\begin{array}{ccc|c} 3 & 4 & 1 & 38 \\ 5 & -3 & 7 & 47 \\ 7 & -2 & -3 & 0 \end{array}\right) \xrightarrow[7I-3III]{5I-3II} \left(\begin{array}{ccc|c} 3 & 4 & 1 & 38 \\ 0 & 29 & -16 & 49 \\ 0 & 34 & 16 & 266 \end{array}\right)$

$\xrightarrow{\frac{1}{2}III} \left(\begin{array}{ccc|c} 3 & 4 & 1 & 38 \\ 0 & 29 & -16 & 49 \\ 0 & 17 & 8 & 133 \end{array}\right) \xrightarrow{17II-29III} \left(\begin{array}{ccc|c} 3 & 4 & 1 & 38 \\ 0 & 29 & -16 & 49 \\ 0 & 0 & -504 & -3024 \end{array}\right)$

$\underline{III}: \quad -504x_3 = -3024 \iff \boxed{x_3 = 6}$

$\underline{II}: \quad 29x_2 - 16x_3 = 49 \iff \boxed{x_2 = 5}$

$I: \quad 3x_1 + 4x_2 + x_3 = 38 \iff \boxed{x_1 = 4}$

---

c) $\left(\begin{array}{cccc|c} 1 & 3 & -4 & -2 & -3 \\ 1 & 4 & 1 & -1 & 13 \\ 3 & 8 & -4 & -3 & 10 \\ 2 & 7 & -3 & 1 & 6 \end{array}\right) \xrightarrow[\substack{3I-III \\ 2I-IV}]{I-II} \left(\begin{array}{cccc|c} 1 & 3 & -4 & -2 & -3 \\ 0 & -1 & -5 & -1 & -16 \\ 0 & 1 & -8 & -3 & -19 \\ 0 & -1 & -5 & -5 & -17 \end{array}\right)$

$\xrightarrow[II-IV]{II+III} \left(\begin{array}{cccc|c} 1 & 3 & -4 & -2 & -3 \\ 0 & -1 & -5 & -1 & -16 \\ 0 & 0 & -13 & -4 & -35 \\ 0 & 0 & 0 & 4 & -4 \end{array}\right)$

$\underline{IV}: \quad 4x_4 = -4 \iff \boxed{x_4 = -1}$

$\underline{III}: \quad -13x_3 - 4x_4 = -35 \iff \boxed{x_3 = 3}$

$\underline{II}: \quad x_2 + 5x_3 + x_4 = 16 \iff \boxed{x_2 = 2}$

$I: \quad x_1 + 3x_2 - 4x_3 - 2x_4 = -3 \iff \boxed{x_1 = 1}$

---

d) $\left(\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 1 & -1 & 3 & 8 \\ 1 & 2 & -1 & 2 \\ 2 & 1 & -3 & 1 \end{array}\right) \xrightarrow[\substack{I-III \\ 2I-IV}]{I-II} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 2 & -2 & -2 \\ 0 & -1 & 2 & 4 \\ 0 & 1 & 5 & 11 \end{array}\right)$

$\xrightarrow{\frac{1}{2}II} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & -1 & -1 \\ 0 & -1 & 2 & 4 \\ 0 & 1 & 5 & 11 \end{array}\right) \xrightarrow[II-IV]{II+III} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & -6 & -12 \end{array}\right)$

$\xrightarrow{-\frac{1}{6}IV} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 1 & 2 \end{array}\right) \xrightarrow{III-IV} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{array}\right)$

$\underline{IV}: \quad 0 = 1 \ (f) \implies$ es gibt keine Lösung!

84 a) $\left(\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 1 & -1 & 3 & 8 \\ 3 & 1 & 5 & 20 \end{array}\right) \xrightarrow[3I-III]{I-II} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 2 & -2 & -2 \\ 0 & 2 & -2 & -2 \end{array}\right)$

$\xrightarrow[II-III]{\frac{1}{2}II} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right) \Rightarrow x_2 - x_3 = -1$

setze $\boxed{x_3 = \lambda} \in \mathbb{R}$; $II \Rightarrow \boxed{x_2 = \lambda - 1}$

$I: \; x_1 + x_2 + x_3 = 6 \iff \boxed{x_1 = 7 - 2\lambda}$

Lösungsmenge = Schnittgerade dreier Ebenen:

$$\vec{x} = \begin{pmatrix} 7-2\lambda \\ -1+\lambda \\ \lambda \end{pmatrix} = \boxed{\begin{pmatrix} 7 \\ -1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}} \; ; \lambda \in \mathbb{R}$$

homogene Lösung $\mathbb{L}_0 = \lambda \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix} ; \lambda \in \mathbb{R}$

b) $\left(\begin{array}{ccc|c} 2 & 3 & -4 & 25 \\ -6 & 8 & -4 & 8 \\ 3 & -4 & 2 & -4 \end{array}\right) \xrightarrow[3I-2III]{3I+II} \left(\begin{array}{ccc|c} 2 & 3 & -4 & 25 \\ 0 & 17 & -16 & 83 \\ 0 & 17 & -16 & 83 \end{array}\right)$

$\xrightarrow{II-III} \left(\begin{array}{ccc|c} 2 & 3 & -4 & 25 \\ 0 & 17 & -16 & 83 \\ 0 & 0 & 0 & 0 \end{array}\right)$; $II: \; 17x_2 - 16x_3 = 83$

setze $\boxed{x_3 = 17\lambda}$ mit $\lambda \in \mathbb{R}$; $II \Rightarrow \boxed{x_2 = \frac{83}{17} + 16\lambda}$

$I: \; 2x_1 + 3x_2 - 4x_3 = 25 \iff \boxed{x_1 = \frac{88}{17} + 10\lambda}$

$$\vec{x} = \begin{pmatrix} 88/17 + 10\lambda \\ 83/17 + 16\lambda \\ 17\lambda \end{pmatrix} = \boxed{\frac{1}{17}\begin{pmatrix} 88 \\ 83 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 10 \\ 16 \\ 17 \end{pmatrix}} \; ; \lambda \in \mathbb{R}$$

homogene Lösung $\lambda \begin{pmatrix} 10 \\ 16 \\ 17 \end{pmatrix} ; \lambda \in \mathbb{R}$

85a) $\left(\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 1 & -1 & 3 & 8 \\ 3 & 1 & 5 & 10 \end{array}\right) \xrightarrow[3I-III]{I-II} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 2 & -2 & -2 \\ 0 & 2 & -2 & 8 \end{array}\right)$

$\xrightarrow[\frac{1}{2}III]{\frac{1}{2}II} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & -1 & -1 \\ 0 & 1 & -1 & 4 \end{array}\right) \xrightarrow{II-III} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 0 & -5 \end{array}\right)$

$III: \; 0 = -5 \; (f) \Rightarrow$ es gibt keine Lösung

homogene Lsg: $II: \; x_2 - x_3 = 0$, setze $\boxed{x_3 = \lambda} \Rightarrow \boxed{x_2 = \lambda}$

$I: \; x_1 + x_2 + x_3 = 0 \iff \boxed{x_1 = -2\lambda} \Rightarrow \boxed{\mathbb{L}_0 = \left\{ \lambda \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix} \middle| \lambda \in \mathbb{R} \right\}}$

b) $\left(\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 1 & -1 & 3 & 8 \\ 1 & 2 & -1 & 2 \\ 2 & 1 & -3 & 1 \end{array}\right) \xrightarrow[\substack{I-III \\ 2I-IV}]{I-II} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 2 & -2 & -2 \\ 0 & -1 & 2 & 4 \\ 0 & 1 & 5 & 11 \end{array}\right)$

$\xrightarrow{\frac{1}{2}II} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & -1 & -1 \\ 0 & -1 & 2 & 4 \\ 0 & 1 & 5 & 11 \end{array}\right) \xrightarrow[II-IV]{II+III} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & -6 & -12 \end{array}\right)$

$\xrightarrow{6III+IV} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 6 \end{array}\right)$; $IV: \ 0 = 6 \ (\not{f})$
$\Rightarrow$ keine Lösung

homogene Lösung $III: \boxed{x_3 = 0}$; $II \ x_2 - x_3 = 0 \Leftrightarrow \boxed{x_2 = 0}$
$I: \ x_1 + x_2 + x_3 = 0 \Leftrightarrow \boxed{x_1 = 0}$

$\Rightarrow \boxed{\vec{x} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}}$

---

86 $\left(\begin{array}{ccc|c} 2 & 2t & 3t+1 & 1 \\ -1 & -t & -2t & 3 \\ 1 & 2t & t+1 & 4 \end{array}\right) \xrightarrow[I \text{ mit } III]{\text{tausche}} \left(\begin{array}{ccc|c} 1 & 2t & t+1 & 4 \\ -1 & -t & -2t & 3 \\ 2 & 2t & 3t+1 & 1 \end{array}\right)$

$\xrightarrow[2I-III]{I+II} \left(\begin{array}{ccc|c} 1 & 2t & t+1 & 4 \\ 0 & t & 1-t & 7 \\ 0 & 2t & 1-t & 7 \end{array}\right) \xrightarrow{2II-III} \left(\begin{array}{ccc|c} 1 & 2t & t+1 & 4 \\ 0 & t & 1-t & 7 \\ 0 & 0 & 1-t & 7 \end{array}\right)$

$III: \ (1-t)x_3 = 7 \Rightarrow$ für $\underline{t=1}$ keine Lösung,
da $0 = 7 \ (\not{f})$

Für $t \neq 1$ gibt es Lösungen:

Sei $\underline{t \neq 0} \overset{III}{\Rightarrow} \boxed{x_3 = \frac{7}{1-t}}$; $II: \ tx_2 + (1-t)x_3 = 7$
$\underset{t \neq 0}{\Leftrightarrow} \boxed{x_2 = 0}$

$I: \ x_1 + (t+1)x_3 = 4$
$\Leftrightarrow \boxed{x_1 = 4 - \frac{7(t+1)}{1-t}}$

Sei $\underline{t = 0} \overset{III}{\Rightarrow} \boxed{x_3 = 7}$; $I: \ x_1 + x_3 = 4 \Leftrightarrow \boxed{x_1 = -3}$
x_2 frei wählbar ; $\boxed{x_2 = \lambda} \in \mathbb{R}$

$\boxed{\vec{x} = \begin{pmatrix} -3 \\ \lambda \\ 7 \end{pmatrix} = \begin{pmatrix} -3 \\ 0 \\ 7 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}}$; $\lambda \in \mathbb{R}$

Zusammenfassend:

a) keine Lösung für $t = 1$
b) genau eine Lösung für $t \notin \{0; 1\}$
c) ∞ viele Lösungen für $t = 0$